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**On the transverse scalar curvature of a compact Sasaki manifold.** (English) Zbl 1323.53047

Complex Manifolds 1, 52-63 (2014).

Summary: We show that the standard picture regarding the notion of stability of constant scalar curvature metrics in Kähler geometry described by S. K. Donaldson, which involves the geometry of infinite dimensional groups and spaces, can be applied to the constant scalar curvature metrics in Sasaki geometry with only few modification. We prove that the space of Sasaki metrics is an infinite dimensional symmetric space and that the transverse scalar curvature of a Sasaki metric is a moment map of the strict contactomophism group.

**MSC:**

53C25 Special Riemannian manifolds (Einstein, Sasakian, etc.)
53C35 Differential geometry of symmetric spaces
58B25 Group structures and generalizations on infinite-dimensional manifolds



**Keywords:**

transverse scalar curvature; symmetric space; moment map

**Full Text:** DOI arXiv

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