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Distribution of integral lattice points in an ellipsoid with a Diophantine center. (English)
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J. Number Theory 157, 468-506 (2015).

In this paper, the authors present some asymptotic values of the normalised deviations of the number of lattice points inside a rational ellipsoid $E_t^M(\alpha) \in \mathbb{R}^n$, with centre α. Let

$$E_t^M(\alpha) = \left\{x \in \mathbb{R}^n : Q_M(x-\alpha) \leq t^2\right\}, \qquad t \in \mathbb{R}_{>0},$$

where Q_M is the quadratic form with corresponding positive definitive symmetric $n \times n$ matrix M, and where the central vector α is said to be of "Diophantine type κ" if there exists a constant $c_0 > 0$, such that

$$\left|\alpha - \frac{m}{q}\right| > \frac{c_0}{q^\kappa} \quad \text{for all } m \in \mathbb{Z}^n, \text{ and } q \in \mathbb{N}.$$

Then denoting by $N_M(t) = \#\left\{\mathbb{Z}^n \cap E_t^M(\alpha)\right\}$, the number of lattice points inside the ellipsoid $E_t^M(\alpha)$, and by $|E_t^M|$ the n-dimensional volume of the ellipsoid E_t^M (which is independent of the choice of α), the authors consider the asymptotics of the normalised deviation $F_M(t)$ defined by

$$F_M(t) = \frac{N_M(t) - \left|E_1^M\right| t^n}{t^{(n-1)/2}}, \qquad \text{as } t \to \infty,$$

as well as the normalised deviation $S_M(t,\eta)$ of the number of lattice points inside the shell between the elliptic spheres of radii t and $t+\eta$ given by

$$S_M(t,\eta) = \frac{N_M(t+\eta) - N_M(t) - \left|E_1^M\right| ((t+\eta)^n - t^n)}{\sqrt{\eta}\; t^{(n-1)/2}}, \qquad \text{as } t \to \infty \text{ and as } \eta \to 0.$$

For $\eta \geq 0$, and provided that there exists some $L > 0$ such that $T^{-L} < \eta \ll 1$, it is shown (Theorem 1.2) that

$$\lim_{T\to\infty} \langle F_M \rangle_T = 0, \qquad \text{and} \qquad \lim_{T\to\infty} \langle S_M(\cdot\,,\eta)\rangle_T = 0.$$

With the constraints $n \geq 2$; $\alpha \in \mathbb{R}^n$ a vector of Diophantine type $\kappa < (n-1)/(n-2)$; $(\alpha, 1) \in \mathbb{R}^{n+1}$ a vector whose components are linearly independent over $\mathbb{Q}$; $M = \operatorname{diag}(a_1, \dots a_n)$ a diagonal matrix with entries in $\mathbb{N}$, and $\eta \gg T^{-\gamma}$ for some $\gamma \in (0,1)$, it is also shown (Theorem 1.5) that

$$\lim_{T\to\infty} \langle |S_M(\cdot\,,\eta)|^2\rangle_T = n|E_1^M|, \qquad \text{as } \mu \to 0.$$

In translating the ellipsoid from the origin to a Diophantine vector, exponential sums appear in the expansion of the counting function $N_M(t)$. The mean square limits of these exponential sums are considered, yielding further results and extending a previous result of *J. Marklof* on Euclidean balls to some ellipsoids [Acta Arith. 117, No. 4, 353–370 (2005; Zbl 1075.11023)].

Reviewer: Matthew C. Lettington (Cardiff)

MSC:

11P21 Lattice points in specified regions
11H06 Lattices and convex bodies (number-theoretic aspects)
11L07 Estimates on exponential sums
37C40 Smooth ergodic theory, invariant measures for smooth dynamical systems
42B05 Fourier series and coefficients in several variables

Keywords:

lattice points; convex bodies; rational ellipsoids; exponential sums

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References:

[1] Bleher, P.; Bourgain, J., Distribution of the error term for the number of lattice points inside a shifted ball, (Analytic Number Theory, vol. 1, Allerton Park, IL, 1995, Progr. Math., vol. 3, (1996), Birkhäuser), 141-153 · Zbl 0866.11056

[2] Bleher, P.; Dyson, F., Mean square value of exponential sums related to representation of integers as sum of two squares, Acta Math., 68, 3, 71-84, (1994) · Zbl 0915.11048

[3] Kang, H.; Sobolev, A. V., Distribution of integer lattice points in a ball centred at a Diophantine point, Mathematika, 56, 118-134, (2010) · Zbl 1252.11075

[4] Lion, G.; Vergne, M., The Weil representation, Maslov index and theta series, Progr. Math., vol. 6, (1980), Birkhäuser · Zbl 0444.22005

[5] Marklof, J., Pair correlation densities of inhomogeneous quadratic forms, II, Duke Math. J., 115, 409-434, (2002) · Zbl 1136.11325

[6] Marklof, J., Pair correlation densities of inhomogeneous quadratic forms, Ann. of Math. (2), 158, 419-471, (2003) · Zbl 1106.11018

[7] Marklof, J., Mean square value of exponential sums related to the representation of integers as sums of squares, Acta Arith., 117, 353-370, (2005) · Zbl 1075.11023

[8] Schmidt, W. M., Approximation to algebraic numbers, Enseign. Math. (2), 17, 187-253, (1971) · Zbl 0226.10033