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Neighbor sum distinguishing total colorings of triangle free planar graphs. (English)

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Summary: A total k-coloring c of a graph G is a proper total coloring c of G using colors of the set $[k] = \{1, 2, \ldots, k\}$. Let $f(u)$ denote the sum of the color on a vertex u and colors on all the edges incident to u. A k-neighbor sum distinguishing total coloring of G is a total k-coloring of G such that for each edge $uv \in E(G)$, $f(u) \neq f(v)$. By $\chi''_{\text{nsd}}(G)$, we denote the smallest value k in such a coloring of G. *M. Pilśniak* and *M. Woźniak* [Graphs Comb. 31, No. 3, 771–782 (2015; Zbl 1312.05054)] conjectured that $\chi''_{\text{nsd}}(G) \leq \Delta(G)+3$ for any simple graph with maximum degree $\Delta(G)$. In this paper, by using the famous Combinatorial Nullstellensatz, we prove that the conjecture holds for any triangle free planar graph with maximum degree at least 7.

MSC:

05C15 Coloring of graphs and hypergraphs

Keywords:

neighbor sum distinguishing total coloring; combinatorial Nullstellensatz; triangle free planar graph

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