

**Kucharczyk, Robert A.****Modular embeddings and rigidity for Fuchsian groups.** (English) Zbl 1325.20044

Acta Arith. 169, No. 1, 77-100 (2015).

Summary: We prove a rigidity theorem for semiarithmetic Fuchsian groups: If  $\Gamma_1, \Gamma_2$  are two semiarithmetic lattices in  $\mathrm{PSL}(2, \mathbb{R})$  virtually admitting modular embeddings, and  $f: \Gamma_1 \rightarrow \Gamma_2$  is a group isomorphism that respects the notion of congruence subgroups, then  $f$  is induced by an inner automorphism of  $\mathrm{PGL}(2, \mathbb{R})$ .

Reviewer: [Reviewer \(Berlin\)](#)**MSC:**

[20H10](#) Fuchsian groups and their generalizations (group-theoretic aspects)  
[11F06](#) Structure of modular groups and generalizations; arithmetic groups  
[22E40](#) Discrete subgroups of Lie groups  
[14G35](#) Modular and Shimura varieties

Cited in **3** Documents**Keywords:**

semiarithmetic Fuchsian groups; rigidity; modular embeddings; congruence subgroups; arithmetic groups; semiarithmetic groups; semiarithmetic lattices

**Software:**[MathOverflow](#)**Full Text:** [DOI](#) [arXiv](#)**References:**

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