Chen, Yong-Gao; **Fang, Jin-Hui**

On a conjecture of Sárközy and Szemerédi. (English) Zbl 1370.11017

Acta Arith. 169, No. 1, 47-58 (2015).

Summary: Two infinite sequences A and B of non-negative integers are called *infinite additive complements* if their sum contains all sufficiently large integers. In 1994, *Sárközy* and *E. Szemerédi* [Acta Math. Hung. 64, No. 3, 237–245 (1994; Zbl 0816.11013)] conjectured that there exist infinite additive complements A and B with $\limsup A(x)B(x)/x \le 1$ and $A(x)B(x) - x = O(\min\{A(x), B(x)\})$, where $A(x)$ and $B(x)$ are the counting functions of A and B, respectively. We prove that, for infinite additive complements A and B, if $\limsup A(x)B(x)/x \le 1$, then, for any given $M > 1$, we have

$$A(x)B(x) - x \ge (\min\{A(x), B(x)\})^M$$

for all sufficiently large integers x. This disproves the above Sárközy-Szemerédi conjecture. We also pose several problems for further research.

MSC:

11B13 Additive bases, including sumsets
11B34 Representation functions

Keywords:

additive complements; sequences; counting functions

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