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On using the hypervolume indicator to compare Pareto fronts: applications to multi-criteria optimal experimental design. (English) Zbl 1311.62115

J. Stat. Plann. Inference 160, 60-74 (2015).

Summary: The Pareto approach to optimal experimental design simultaneously considers multiple objectives by constructing a set of Pareto optimal designs while explicitly considering trade-offs between opposing criteria. Various algorithms have been proposed to populate Pareto fronts of designs, and evaluating and comparing these fronts-and by extension the algorithms that produce them-is crucial. In this paper, we first propose a framework for comparing algorithm-generated Pareto fronts based on a refined hypervolume indicator. We then theoretically address how the choice of the reference point affects comparisons of Pareto fronts, and demonstrate that our approach is Pareto compliant. Based on our theoretical investigation, we provide rules for choosing reference points when two-dimensional Pareto fronts are compared. Because theoretical results for three-dimensional fronts are difficult to obtain, we propose an empirical rule for the three-dimensional case by making an analogy to the rules for two dimensions. We also consider the use of our procedure in evaluating the progress of a front-constructing algorithm, and illustrate our work with two examples from the literature.

MSC:

[62K05](#) Optimal statistical designs

[90C29](#) Multi-objective and goal programming

Cited in 4 Documents

Keywords:

[Pareto front](#); [multi-objective optimization](#); [design of experiments](#); [point exchange](#)

Software:

[HypE](#); [NBI](#); [SMS-EMOA](#); [weightedHypervolume](#)

Full Text: [DOI](#)

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