Flannery, D. L.; Egan, R.
On linear shift representations. (English) Zbl 1346.20071
J. Pure Appl. Algebra 219, No. 8, 3482-3494 (2015).

From the text: We introduce and develop the concept of (linear) *shift representation*. This derives from a certain action on 2-cocycle groups that preserves both cohomological equivalence and orthogonality for cocyclic designs, discovered by K. J. Horadam. Detailed information about fixed point spaces and reducibility is given. We also discuss results of computational experiments, including the calculation of shift orbit structure and searching for orthogonal cocycles.

We now summarize the content of the paper. In Section 2 we prove elementary facts about shift representations. The main results of Section 3 are a determination of fixed points under shift action in the full cocycle space, and a bound on the dimension of the fixed coboundary space. We thereby solve most of Research Problem 55 (1) in [*K. J. Horadam*, Hadamard matrices and their applications. Princeton: Princeton University Press (2007; Zbl 1145.05014)]. Some relevant linear group theory is then given in Section 4. This serves as background for Section 5, where we establish that a shift representation is hardly ever completely reducible. In fact, we provide criteria for deciding irreducibility and complete reducibility. As an illustration of the practical nature of our approach, in the final section we describe new results obtained from our `Magma` [*W. Bosma* et al., J. Symb. Comput. 24, No. 3-4, 235-265 (1997; Zbl 0898.68039)] implementation of procedures to compute with shift representations. Open questions arising from the computational work are posed.

MSC:

Cited in **2** Documents

20J05 Homological methods in group theory
05B20 Combinatorial aspects of matrices (incidence, Hadamard, etc.)
20C15 Ordinary representations and characters
20H30 Other matrix groups over finite fields

Keywords:

shift representations; 2-cocycle groups; cohomological equivalence; orthogonal cocyclic designs; fixed point spaces; orthogonal cocycles; linear groups; irreducibility; complete reducibility

Software:

Magma

Full Text: DOI

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