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**Complexity analysis of hypergeometric orthogonal polynomials.** (English) Zbl 1351.60036  
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Summary: The complexity measures of the Crámer-Rao, Fisher-Shannon and LMC (López-Ruiz, Mancini and Calvet) types of the Rakhmanov probability density  $\rho_n(x) = \omega(x)p_n^2(x)$  of the polynomials  $p_n(x)$  orthogonal with respect to the weight function  $\omega(x)$ ,  $x \in (a, b)$ , are used to quantify various two-fold facets of the spreading of the Hermite, Laguerre and Jacobi systems all over their corresponding orthogonality intervals in both analytical and computational ways. Their explicit (Crámer-Rao) and asymptotical (Fisher-Shannon, LMC) values are given for the three systems of orthogonal polynomials. Then, these complexity-type mathematical quantities are numerically examined in terms of the polynomial's degree  $n$  and the parameters which characterize the weight function. Finally, several open problems about the generalized hypergeometric functions of Lauricella and Srivastava-Daoust types, as well as on the asymptotics of weighted  $L_q$ -norms of Laguerre and Jacobi polynomials are pointed out.

**MSC:**

- 60F99 Limit theorems in probability theory
- 42C05 Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis
- 33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)

**Keywords:**

hypergeometric orthogonal polynomials; information theory; Fisher-Shannon complexity; LMC complexity; asymptotics; Crámer-Rao complexity

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