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Existence of knotted vortex tubes in steady Euler flows. (English) Zbl 1317.35184

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The purpose of this paper is the proof of a theorem which states that knotted and linked thin vortex tubes for steady solutions to the incompressible Euler equation in $\mathbb{R}^3$ exist. The proof consists of three steps, which are gradually improved in the paper.

- The construction of a local Beltrami field, which satisfies the Beltrami equation curl $v = \lambda v$, and has a set of certain invariant tori.
- It is proved that these invariant tori are "robust" in a certain sense.
- It is proved that the local Beltrami field can be approximated by a global field, which satisfies the Beltrami equation in $\mathbb{R}^3$, and drops off at infinity in an optimal way.

The proofs use Lyapunov stability, Cauchy-Schwartz, Poincaré, Jensen and Sobolev inequalities, Hodge decomposition, Riesz representation theorem, Fredholm alternative, Poincaré map, Hahn-Banach theorem, Riesz-Markov theorem, Poissons equation and spherical Bessel functions. Finally, applications to Navier-Stokes equation are briefly discussed.

Reviewer: Thomas Ernst (Uppsala)

MSC:

35Q31	Euler equations
37N10	Dynamical systems in fluid mechanics, oceanography and meteorology
57M25	Knots and links in the 3-sphere (MSC2010)
35J25	Boundary value problems for second-order elliptic equations
35J40	Boundary value problems for higher-order elliptic equations

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Euler equation; invariant tori; KAM theory; knots; Beltrami fields; Runge-type approximation

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