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Constructing entire functions by quasiconformal folding. (English) Zbl 1338.30016
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For a finite plane tree T , a polynomial p is associated with only two critical values ± 1 such that $T_p = p^{-1}([-1, 1])$ is a plane tree which is equivalent to T . To an infinite plane tree T , there corresponds a certain entire function f . In order to explain this in more detail, let us consider the singular set $S(f)$ of f which is the closure of the critical values and finite asymptotic values of f . The Speiser class \mathcal{S} is the set of transcendental entire functions with a finite singular set. Furthermore, let $\mathcal{S}_n \subset \mathcal{S}$ be those functions with at most n singular values and $\mathcal{S}_{p,q}$ be those functions with p critical values and q finite asymptotic values. Finally, let \mathcal{B} denote the Eremenko-Lyubich class of transcendental entire functions with bounded (not necessarily finite) singular set.

Now, for an infinite plane tree T with certain mild geometric conditions the author develops a method to construct a corresponding entire function f in the class $\mathcal{S}_{2,0}$ with the only critical values ± 1 such that $T_f = f^{-1}([-1, 1])$ is a plane tree which approximates T in a precise way. His method uses quasiconformal mappings and the measurable Riemann mapping theorem. Furthermore, he applies his method to solve a number of open problems, e.g., the area conjecture of Eremenko and Lyubich and the existence of a function in \mathcal{B} whose Fatou set has a wandering domain.

Reviewer: [Rainer Brück \(Dortmund\)](#)

MSC:

- [30C65](#) Quasiconformal mappings in \mathbb{R}^n , other generalizations
- [30D15](#) Special classes of entire functions of one complex variable and growth estimates
- [30D05](#) Functional equations in the complex plane, iteration and composition of analytic functions of one complex variable

Cited in **4** Reviews
Cited in **20** Documents

Keywords:

quasiconformal maps; entire functions; Speiser class; Eremenko-Lyubich class; bounded singular set; finite singular set; wandering domains; area conjecture

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