Gérard, C.; **Wrochna, M.**

Hadamard states for the linearized Yang-Mills equation on curved spacetime. (English)

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Summary: We construct Hadamard states for the Yang-Mills equation linearized around a smooth, space-compact background solution. We assume the spacetime is globally hyperbolic and its Cauchy surface is compact or equal $\mathbb{R}^d$.

We first consider the case when the spacetime is ultra-static, but the background solution depends on time. By methods of pseudodifferential calculus we construct a parametrix for the associated vectorial Klein-Gordon equation. We then obtain Hadamard two-point functions in the gauge theory, acting on Cauchy data. A key role is played by classes of pseudodifferential operators that contain microlocal or spectral type low-energy cutoffs.

The general problem is reduced to the ultra-static spacetime case using an extension of the deformation argument of *S. A. Fulling* et al. [Ann. Phys. 136, 243–272 (1982; Zbl 0495.35054)].

As an aside, we derive a correspondence between Hadamard states and parametrices for the Cauchy problem in ordinary quantum field theory.

MSC:

83C47	Methods of quantum field theory in general relativity and gravitational theory
81T13	Yang-Mills and other gauge theories in quantum field theory
70S15	Yang-Mills and other gauge theories in mechanics of particles and systems
83C05	Einstein's equations (general structure, canonical formalism, Cauchy problems)
35L05	Wave equation
81T20	Quantum field theory on curved space or space-time backgrounds

Keywords:

Hadamard states; linearized Yang-Mills equation; curved spacetime; Klein-Gordon equation; Cauchy data

References:

[1] Araki, H.; Shiraishi, M., On quasi-free states of canonical commutation relations I, Publ. RIMS Kyoto Univ., 7, 105-120, (1971) · Zbl 0239.46066

[2] Benini, M.: Optimal Space of Linear Classical Observables for Maxwell k-Forms Via Spacelike and Timelike Compact de Rham Cohomologies. arXiv:1401.7563 (2014) · Zbl 1381.83045

[3] Bär, C., Ginoux, N.: Classical and Quantum Fields on Lorentzian Manifolds. Global Differential Geometry. Springer, Berlin (2012) · Zbl 0938.81028

[4] Bär, C., Ginoux, N., Pfäffle, F.: Wave equation on Lorentzian manifolds and quantization. In: ESI Lectures in Mathematics and Physics, EMS (2007) · Zbl 0239.46066

[5] Choquet-Bruhat, Y.: Yang-Mills Fields on Lorentzian Manifolds. Mechanics, Analysis and Geometry: 200 Years After Lagrange. North-Holland Delta Series, North Holland (1991) · Zbl 1161.81022

[6] Choquet-Bruhat, Y.; Christodoulou, D., Existence of global solutions of the Yang-Mills, Higgs and spinor fields equations in 3 + 1 dimensions, Ann. Sci. École Norm. Sup., 14, 481-506, (1981) · Zbl 0499.35076

[7] Cycon H.L., Froese R., Kirsch W., Simon B.: Schrödinger Operators with Applications to Quantum Mechanics and Global Geometry. Springer, New York (1987) · Zbl 0619.47005

[8] Chruściel, P.T.; Shatah, J., Global existence of solutions of the Yang-Mills equations on globally hyperbolic four dimensional Lorentzian manifolds, Asian J. Math., 1, 530-548, (1997) · Zbl 0912.58039

[9] Dereziński, J.: Quantum Fields with Classical Perturbations. arXiv:1307.1162 (2013) · Zbl 0912.58039

[10] Dütsch, M.; Fredenhagen, K., A local (perturbative) construction of observables in gauge theories: the example of QED, Commun. Math. Phys., 203, 71, (1999) · Zbl 0938.81028

[11] Dereziński J., Gérard C.: Mathematics of quantization and quantum fields. Cambridge Monographs in Mathematical Physics. Cambridge University Press, Cambridge (2013)

[12] Dimock, J., Dirac quantum fields on a manifold, Trans. Am. Math. Soc., 269, 133-147, (1982) · Zbl 0518.58018

[13] Dimock, J., Quantized electromagnetic field on a manifold, Rev. Math. Phys., 4, 223-233, (1992) · Zbl 0760.53049

[14] Dappiaggi, C.; Hack, T.-P.; Sanders, K., Electromagnetism, local covariance, the Aharonov-Bohm effect and gauss' law, Commun. Math. Phys., 328, 625-667, (2014) · Zbl 1293.81034

[15] Dappiaggi, C.; Siemssen, D., Hadamard states for the vector potential on asymptotically flat spacetimes, Rev. Math. Phys., 25, 1350002, (2013) · Zbl 1278.81134

[16] Fulling, S.A.; Narcowich, F.J.; Wald, R.M., Singularity structure of the two-point function in quantum field theory in curved spacetime, II, Ann. Phys., 136, 243-272, (1981) · Zbl 0495.35054

[17] Fewster, C.J.; Pfenning, M.J., A quantum weak energy inequality for spin-one fields in curved spacetime, J. Math. Phys., 44, 4480, (2003) · Zbl 1062.81115

[18] Finster, F., Strohmaier, A.: Gupta-Bleuler Quantization of the Maxwell Field in Globally Hyperbolic Space-Times. arXiv:1307.1632 (2013) · Zbl 1321.81063

[19] Furlani, E.P., Quantization of the electromagnetic field on static spacetimes, J. Math. Phys., 36, 1063-1079, (1995) · Zbl 0822.53048

[20] Furlani, E.P., Quantization of massive vector fields in curved spacetime, J. Math. Phys., 40, 2611, (1999) · Zbl 0961.81058

[21] Gérard, C.; Wrochna, M., Construction of Hadamard states by pseudo-differential calculus, Commun. Math. Phys., 325, 713-755, (2014) · Zbl 1298.81214

[22] Hack, T.-P., Schenkel, A.: Linear bosonic and fermionic quantum gauge theories on curved spacetimes. Gen. Rel. Grav. 1-34 (2012) · Zbl 0499.35076

[23] Hollands, S.: The hadamard condition for dirac fields and adiabatic states on Robertson-Walker spacetimes. Commun. Math. Phys. \textbf{216}, 635-661 (2001) · Zbl 0976.58023

[24] Hollands, S., Renormalized quantum Yang-Mills fields in curved spacetime, Rev. Math. Phys., 20, 1033-1172, (2008) · Zbl 1161.81022

[25] Hörmander L.: The analysis of linear partial differential operators I. Distribution Theory and Fourier Analysis. Springer, Berlin (1985)

[26] Junker, W.: Adiabatic Vacua and Hadamard States for Scalar Quantum Fields on Curved Spacetime. PhD thesis, University of Hamburg 1995 · Zbl 0869.53053

[27] Khavkine, I.: Characteristics, Conal Geometry and Causality in Locally Covariant Field Theory. arXiv:1211.1914 (2012) · Zbl 0518.58018

[28] Marathe K.B., Martucci G.: Mathematical foundations of gauge theories. Studies in Mathematical Physics, vol. 5. North-Holland, Amsterdam (1992) · Zbl 0920.58079

[29] Mühlhoff, R., Cauchy problem and green's functions for first order differential operators and algebraic quantization, J. Math. Phys., 52, 022303, (2011) · Zbl 1314.81116

[30] Pfenning, M.J., Quantization of the Maxwell field in curved spacetimes of arbitrary dimension, Class. Quantum Grav., 26, 135017, (2009) · Zbl 1171.83356

[31] Rejzner, K.: Remarks on Local Gauge Invariance in Perturbative Algebraic Quantum Field Theory. arXiv:1301.7037 (2013) · Zbl 1306.81137

[32] Segal, I., The Cauchy problem for the Yang-Mills equations, J. Funct. Anal., 33, 175-194, (1979) · Zbl 0416.58027

[33] Sahlmann, H.; Verch, R., Microlocal spectrum condition and Hadamard form for vector-valued quantum fields in curved spacetime, Rev. Math. Phys., 13, 1203-1246, (2001) · Zbl 1029.81053

[34] Taylor M.: Pseudo-Differential Operators and Nonlinear PDE. Birkhäuser, Basel (1991)

[35] Wrochna, M., Quantum field theory in static external potentials and Hadamard states, Ann. Henri Poincaré, 13, 1841-1871, (2012) · Zbl 1256.81041

[36] Wrochna, M.: Singularities of Two-Point Functions in Quantum Field Theory. PhD thesis, University of Göttingen (2013)

[37] Wrochna, M., Zahn, J.: Classical Phase Space and Hadamard States in the BRST Formalism for Gauge Field Theories on Curved Spacetime. arXiv:1407.8079 (2014) · Zbl 1447.81165

[38] Zahn, J., The renormalized locally covariant Dirac field, Rev. Math. Phys., 26, 1330012, (2014) · Zbl 1287.81086