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Global smooth solutions to the n -dimensional damped models of incompressible fluid mechanics with small initial datum. (English) Zbl 1311.35236

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Summary: In this paper, we consider the n -dimensional ($n \geq 2$) damped models of incompressible fluid mechanics in Besov spaces and establish the global (in time) regularity of classical solutions provided that the initial data are suitable small.

MSC:

- 35Q35 PDEs in connection with fluid mechanics
- 35B35 Stability in context of PDEs
- 35B65 Smoothness and regularity of solutions to PDEs
- 76D03 Existence, uniqueness, and regularity theory for incompressible viscous fluids
- 76W05 Magnetohydrodynamics and electrohydrodynamics
- 35Q86 PDEs in connection with geophysics
- 86A10 Meteorology and atmospheric physics
- 76E06 Convection in hydrodynamic stability

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Keywords:

Boussinesq equations; surface quasi-geostrophic equation; magnetohydrodynamics equations; damping; global regularity; small initial datum

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