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Quasistatic droplets in randomly perforated domains. (English) Zbl 1315.35166
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In this paper the authors study the Hele-Shaw problem in a randomly perforated domain with zero Neumann boundary conditions. By making use of the extending De Giorgi-Nash-Moser type estimates to perforated domains, the authors establish the almost sure non-degenerated growth of the solution near the free boundary, then show that the solutions and their free boundary converge uniformly to those corresponding to a homogeneous and anisotropic Hele-Shaw problem set, as the characteristic scale of the domain goes to zero.

Reviewer: [Cheng He \(Beijing\)](#)

MSC:

- [35Q35](#) PDEs in connection with fluid mechanics
- [76D27](#) Other free boundary flows; Hele-Shaw flows
- [35B27](#) Homogenization in context of PDEs; PDEs in media with periodic structure

Keywords:

[Hele-Shaw problem](#); [Neumann boundary conditions](#); [randomly perforated domain](#); [homogeneous limit](#); [De Giorgi-Nash-Moser type estimates](#)

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References:

- [1] Akcoglu, M.A.; Krengel, U., Ergodic theorems for superadditive processes, J. Reine Angew. Math., 323, 53-67, (1981) · [Zbl 0453.60039](#)
- [2] Alberti, G.; DeSimone, A., Quasistatic evolution of sessile drops and contact angle hysteresis, Arch. Ration. Mech. Anal., 202, 295-348, (2011) · [Zbl 1276.76016](#)
- [3] Barlow, M.T., Random walks on supercritical percolation clusters, Ann. Probab., 32, 3024-3084, (2004) · [Zbl 1067.60101](#)
- [4] Bensoussan A., Lions J.L., Papanicolaou G.: Asymptotic analysis for periodic structures. Studies in Mathematics and its Applications, Vol. 5. North-Holland Publishing Co., Amsterdam (1978) · [Zbl 0404.35001](#)
- [5] Biskup, M., Recent progress on the random conductance model, Probab. Surv., 8, 294-373, (2011) · [Zbl 1245.60098](#)
- [6] Blank, I., Sharp results for the regularity and stability of the free boundary in the obstacle problem, Indiana Univ. Math. J., 50, 1077-1112, (2001) · [Zbl 1032.35170](#)
- [7] Blank, I., Hao, Z.: The mean value theorem for divergence form elliptic operators. arXiv:1302.2952 (2013) · [Zbl 1309.35034](#)
- [8] Caffarelli, L.A., The obstacle problem revisited, J. Fourier Anal. Appl., 4, 383-402, (1998) · [Zbl 0928.49030](#)
- [9] Caffarelli, L.A.; Mellet, A., Random homogenization of an obstacle problem, Ann. Inst. H. Poincaré Anal. Non Linéaire, 26, 375-395, (2009) · [Zbl 1180.35069](#)
- [10] Caffarelli, L.A.; Lee, K.-A.; Mellet, A., Flame propagation in one-dimensional stationary ergodic media, Math. Models Methods Appl. Sci., 17, 155-169, (2007) · [Zbl 1110.76054](#)
- [11] Caffarelli, L.A.; Lee, K.-A.; Mellet, A., Singular limit and homogenization for flame propagation in periodic excitable media, Arch. Ration. Mech. Anal., 172, 153-190, (2004) · [Zbl 1058.76070](#)
- [12] Caffarelli, L.A.; Souganidis, P.E.; Wang, L., Homogenization of fully nonlinear, uniformly elliptic and parabolic partial differential equations in stationary ergodic media, Commun. Pure Appl. Math., 58, 319-361, (2005) · [Zbl 1063.35025](#)
- [13] Choi, S.; Jerison, D.; Kim, I., Regularity for the one-phase Hele-Shaw problem from a Lipschitz initial surface, Am. J. Math., 129, 527-582, (2007) · [Zbl 1189.35384](#)
- [14] Choi, S.; Kim, I., Waiting time phenomena of the Hele-Shaw and the Stefan problem, Indiana Univ. Math. J., 55, 525-552, (2006) · [Zbl 1387.35629](#)
- [15] Cioranescu, D.; Donato, P., Homogénéisation du probleme de Neumann non homogene dans des ouverts perforés, Asymptot. Anal., 1, 115-138, (1988) · [Zbl 0683.35026](#)
- [16] Cioranescu, D.; Paulin, J.S.J., Homogenization in open sets with holes, J. Math. Anal. Appl., 71, 590-607, (1979) · [Zbl 0427.35073](#)

- [17] Dal Maso, G.; Modica, L., Nonlinear stochastic homogenization and ergodic theory, *J. Reine Angew. Math.*, 368, 28-42, (1986) · [Zbl 0582.60034](#)
- [18] De Giorgi, E., Sulla differenziabilità e l'analiticità delle estremali degli integrali multipli regolari, *Mem. Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat.* (3), 3, 25-43, (1957) · [Zbl 0084.31901](#)
- [19] Delmotte, T., Parabolic Harnack inequality and estimates of Markov chains on graphs, *Rev. Mat. Iberoam.*, 15, 181-232, (1999) · [Zbl 0922.60060](#)
- [20] Elliott, C.M.; Janovsky, V., A variational inequality approach to Hele-Shaw flow with a moving boundary, *Proc. R. Soc. Edinb. Sect. A*, 88, 93-107, (1981) · [Zbl 0455.76043](#)
- [21] Evans, L.C.; Gariepy, R.F.: *Measure theory and fine properties of functions*. \textit{Studies in Advanced Mathematics}. CRC Press, Boca Raton, 1992 · [Zbl 0804.28001](#)
- [22] Fabes, E.B.; Stroock, D.W., A new proof of Moser's parabolic Harnack inequality using the old ideas of Nash, *Arch. Ration. Mech. Anal.*, 96, 327-338, (1986) · [Zbl 0652.35052](#)
- [23] Glasner, K.B., A boundary integral formulation of quasi-steady fluid wetting, *J. Comput. Phys.*, 207, 529-541, (2005) · [Zbl 1213.76068](#)
- [24] Grigor'yan, A.A., The heat equation on noncompact Riemannian manifolds, *Mat. Sb.*, 182, 55-87, (1991) · [Zbl 0743.58031](#)
- [25] Grimmett, G.: *Percolation*. \textit{Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]}, Vol. 321, 2nd edn. Springer, Berlin, 1999 · [Zbl 1300.76014](#)
- [26] Grunewald, N.; Kim, I.C., A variational approach to a quasi-static droplet model, *Calc. Var. Partial Differ. Equ.*, 41, 1-19, (2011) · [Zbl 1228.35087](#)
- [27] Hele-Shaw, H.S., Flow of water, *Nature*, 58, 34-36, (1898)
- [28] Jerison, D.; Kim, I., The one-phase Hele-Shaw problem with singularities, *J. Geom. Anal.*, 15, 641-667, (2005) · [Zbl 1155.35496](#)
- [29] Jikov, V.V., Kozlov, S.M., Olenik, O.A.: \textit{Homogenization of Differential Operators and Integral Functionals}. Springer, Berlin, 1994. Translated from the Russian by G. A. Yosifian [G. A. Iosif'yan] · [Zbl 1110.76054](#)
- [30] Kim, I.; Mellet, A., Homogenization of Hele-Shaw problem in periodic and random media, *Arch. Ration. Mech. Anal.*, 194, 507-530, (2009) · [Zbl 1251.76015](#)
- [31] Kim, I.C., Uniqueness and existence results on the Hele-Shaw and the Stefan problems, *Arch. Ration. Mech. Anal.*, 168, 299-328, (2003) · [Zbl 1044.76019](#)
- [32] Kim, I.C., A free boundary problem arising in flame propagation, *J. Differ. Equ.*, 191, 470-489, (2003) · [Zbl 1040.35149](#)
- [33] Kim, I.C., Homogenization of the free boundary velocity, *Arch. Ration. Mech. Anal.*, 185, 69-103, (2007) · [Zbl 1162.35011](#)
- [34] Kim, I.C.; Mellet, A., Homogenization of one-phase Stefan-type problems in periodic and random media, *Trans. Am. Math. Soc.*, 362, 4161-4190, (2010) · [Zbl 1197.35290](#)
- [35] Kinderlehrer, D., Stampacchia, G.: \textit{An Introduction to Variational Inequalities and their Applications}, Vol. 88. Academic Press, Dublin, 1980 · [Zbl 0457.35001](#)
- [36] Kozlov, S.M., Averaging of random structures, *Dokl. Akad. Nauk SSSR*, 241, 1016-1019, (1978)
- [37] Levine, L.; Peres, Y., Scaling limits for internal aggregation models with multiple sources, *J. Anal. Math.*, 111, 151-219, (2010) · [Zbl 1210.82031](#)
- [38] Littman, W.; Stampacchia, G.; Weinberger, H.F., Regular points for elliptic equations with discontinuous coefficients, *Ann. Scuola Norm. Sup. Pisa*, 17, 43-77, (1963) · [Zbl 0116.30302](#)
- [39] Mellet, A.; Nolen, J., Capillary drops on a rough surface, *Interfaces Free Boundaries*, 14, 167-184, (2012) · [Zbl 1300.76014](#)
- [40] Moser, J., A new proof of de Giorgi's theorem concerning the regularity problem for elliptic differential equations, *Commun. Pure Appl. Math.*, 13, 457-468, (1960) · [Zbl 0111.09301](#)
- [41] Papanicolaou G.C., Varadhan S.R.S.: *Boundary value problems with rapidly oscillating random coefficients*. \textit{Random Fields, Vol. I, II (Esztergom, 1979)}, Vol. 27 of \textit{Colloq. Math. Soc. János Bolyai}. North-Holland, Amsterdam, 835-873, 1981 · [Zbl 1276.76016](#)
- [42] Saloff-Coste, L., Uniformly elliptic operators on Riemannian manifolds, *J. Differ. Geom.*, 36, 417-450, (1992) · [Zbl 0735.58032](#)
- [43] Stein, E.M.: *Singular integrals and differentiability properties of functions*. \textit{Princeton Mathematical Series}, Vol. 30. Princeton University Press, Princeton, 1970 · [Zbl 0207.13501](#)
- [44] Zhikov, V.V., Averaging in punctured random domains of general type, *Mat. Zametki*, 53, 41-58, 155, (1993) · [Zbl 0801.60058](#)

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