Zhang, Huiming; **Liu, Yunxiao**; **Li, Bo**

Notes on discrete compound Poisson model with applications to risk theory. (English)

Zbl 1306.60050
Insur. Math. Econ. 59, 325-336 (2014).

Summary: Probability generating function (p.g.f.) is a powerful tool to study discrete compound Poisson (DCP) distribution. By applying inverse Fourier transform of p.g.f., it is convenient to numerically calculate probability density and do parameter estimation. As an application to finance and insurance, we firstly show that in the generalized CreditRisk$^+$ model, the default loss of each debtor and the total default of all debtors are both approximately equal to a DCP distribution, and we give Le Cam's error bound between the total default and a DCP distribution. Next, we consider geometric Brownian motion with DCP jumps and derive its rth moment. We establish the surplus process of the difference of two DCP distributions, and numerically compute the tail probability. Furthermore, we define the discrete pseudo compound Poisson (DPCP) distribution and give the characterizations and examples of DPCP distribution, including the strictly decreasing discrete distribution and the zero-inflated discrete distribution with $P(X = 0) > 0.5$.

MSC:

60G51	Processes with independent increments; Lévy processes
60J75	Jump processes (MSC2010)
91G40	Credit risk
62P05	Applications of statistics to actuarial sciences and financial mathematics

Keywords:

compound Poisson distribution; integer-valued Lévy process; CreditRisk$^+$ model; geometric Brownian motion with jumps; pseudo compound Poisson distribution; Wiener-Lévy theorem

Software:

CreditRisk+

Full Text: DOI

References:

[1] Adelson, R. M., Compound Poisson distributions, Oper. Res., 17, 1, 73-75, (1966)

[2] Barndorff-Nielsen, O. E.; Pollard, D. G.; Shephard, N., Integer-valued Lévy processes and low latency financial econometrics, Quant. Finance, 12, 4, 587-605, (2012) · Zbl 1278.91156

[3] Böttcher, A.; Grudskij, S. M.; Porter, R. M., (Tavidze, A., European Double Barrier Options with a Compound Poisson Component, Progress in Economics Research, vol. 18, (2009), Nova Science Publishers Huntington, New York), 315-331

[4] Buchmann, B.; Grbel, R., Decompounding: an estimation problem for Poisson random sums, Ann. Statist., 31, 4, 1054-1074, (2003) · Zbl 1105.62309

[5] Chan, B., Recursive formulas for discrete distributions, Insurance Math. Econom., 1, 4, 241-243, (1982) · Zbl 0494.62018

[6] Cont, R.; Tankov, P., Financial modelling with jump processes, (2003), Chapman Boca Raton

[7] Credit Suisse Financial Products, 1997. CreditRisk+: A Credit Risk Management Framework. Credit Suisse Financial Products.

[8] Deheuvels, P.; Pfeifer, D., A semigroup approach to Poisson approximation, Ann. Probab., 14, 2, 663-676, (1986) · Zbl 0597.60019

[9] DePril, N., On the exact computation of the aggregate claims distribution in the individual life model, ASTIN Bull., 16, 109-112, (1986)

[10] De Vylder, F. E., Numerical finite-time ruin probabilities by the Picard-lef vre formula, Scand. Actuar. J., 1999, 2, 97-105, (1999) · Zbl 0952.91042

[11] Dhaene, J.; Pril, N. D., On a class of approximative computation methods in the individual risk model, Insurance Math. Econom., 14, 2, 181-196, (1994) · Zbl 0805.62095

[12] Dufresne, F.; Gerber, H. U.; Shiu, E. S.W., Risk theory with the gamma process, ASTIN Bull., 21, 2, 177-192, (1991)

[13] Feller, W., An introduction to probability theory and its applications, vol. I, (1968), Wiley New York · Zbl 0155.23101

[14] Galliher, H. P.; Morse, P. M.; Simond, M., Dynamics of two classes of continuous-review inventory systems, Oper. Res., 7, 3, 362-384, (1959)

[15] Gerber, H. U., Error bounds for the compound Poisson approximation, Insurance Math. Econom., 3, 3, 191-194, (1984) · Zbl 0541.62097

[16] Gmez-Dniz, E.; Sarabia, J. M.; Caldern-Ojeda, E., A new discrete distribution with actuarial applications, Insurance Math. Econom., 48, 3, 406-412, (2011)

[17] (Gundlach, V. M.; Lehrbass, F., CreditRisk+ in the Banking Industry, (2004), Springer Berlin) · Zbl 1046.91001

[18] Haight, F. A., Handbook of the Poisson distribution, (1967), Wiley Los Angeles · Zbl 0152.37706

[19] Hossack, I. B.; Pollard, J. H.; Zehnwirth, B., Introductory statistics with applications in general insurance, (1999), Cambridge University Press London · Zbl 0923.62111

[20] Hürlimann, W., On maximum likelihood estimation for count data models, Insurance Math. Econom., 9, 1, 39-49, (1990) · Zbl 0724.62103

[21] Johnson, N. L.; Kemp, A. W.; Kotz, S., Univariate discrete distributions, (2005), Wiley New Jersey · Zbl 1092.62010

[22] Kaas, R.; Denuit, M.; Dhaene, J.; Goovaerts, M., Modern actuarial risk theory using R, (2008), Springer Berlin · Zbl 1148.91027

[23] Klugman, S. A.; Panjer, H. H.; Willmot, G. E., Loss models: further topics, (2013), Wiley New Jersey · Zbl 1273.62008

[24] Kupper, J., Wahrscheinlichkeitstheoretische modelle in der schadenversicherung, Bl. DGVFM, 5, 4, 451-503, (1962) · Zbl 0108.33002

[25] Lambert, D., Zero-inflated Poisson regression, with an application to defects in manufacturing, Technometrics, 34, 1, 1-14, (1992) · Zbl 0850.62756

[26] Lang, S., Complex analysis, (1999), Springer New York

[27] Le Cam, L., An approximation theorem for the Poisson binomial distribution, Pacific J. Math., 10, 4, 1181-1197, (1960) · Zbl 0118.33601

[28] Lévy, P., Sur LES exponentielles de polynômes et sur l'arithmétique des produits de lois de Poisson, Ann. Sc. Éc. Norm. Supér., 54, 231-292, (1937) · Zbl 63.0499.04

[29] Liang, L.; Tan, D.; Peng, J., Study on the allocation of economic capital of commercial banks with creditrisk+ model, Math. Econ., 22, 3, 221-228, (2005), In Chinese

[30] Merton, R. C., Option pricing when underlying stock returns are discontinuous, J. Financ. Econ., 3, 1, 125-144, (1976) · Zbl 1131.91344

[31] Mikosch, T., Non-life insurance mathematics: an introduction with the Poisson process, (2008), Springer New York

[32] Morales, M.; Schoutens, W., A risk model driven by Lévy processes, Appl. Stoch. Models Bus. Ind., 19, 2, 147-167, (2003) · Zbl 1051.60051

[33] Panjer, H. H., Recursive evaluation of a family of compound distributions, ASTIN Bull., 12, 1, 22-26, (1981)

[34] Parzen, E., Stochastic processes, (1962), Holden-Day San Francisco · Zbl 0107.12301

[35] Patel, Y. C., Estimation of the parameters of the triple and quadruple stuttering-Poisson distributions, Technometrics, 18, 1, 67-73, (1976) · Zbl 0322.62033

[36] Picard, P.; Lefèvre, C., The probability of ruin in finite time with discrete claim size distribution, Scand. Actuar. J., 1997, 1, 58-69, (1997) · Zbl 0926.62103

[37] Roman, S., The formula of faa di bruno, Amer. Math. Monthly, 87, 10, 805-809, (1980) · Zbl 0513.05009

[38] Ross, S. M., An elementary introduction to mathematical finance, (2011), Cambridge University Press London · Zbl 1221.91001

[39] Rulliere, D.; Loisel, S., Another look at the Picard-lefèvre formula for finite-time ruin probabilities, Insurance Math. Econom., 35, 2, 187-203, (2004) · Zbl 1103.91048

[40] Salsburg, D., The lady tasting tea: how statistics revolutionized science in the twentieth century, (2001), WH Freeman New York · Zbl 0981.62002

[41] Sato, K., Lévy processes and infinitely divisible distributions, (2013), Cambridge University Press Cambridge · Zbl 1287.60003

[42] Schoutens, W., Lévy processes in finance: pricing financial derivatives, (2003), Wiley

[43] Schoutens, W.; Cariboni, J., Lévy processes in credit risk, (2009), Wiley · Zbl 1192.91008

[44] Shao, J., Mathematical statistics, (2003), Springer New York · Zbl 1018.62001

[45] Shen, Y.; Yin, C., Optimal dividend problem for a compound Poisson risk model, Appl. Math., 5, 10, 1496-1502, (2014)

[46] Shiu, E. S., Some functional equations in actuarial mathematics, Actuar. Res. Clearing House, 2, 191-208, (1988)

[47] Steele, J. M., Le cam's inequality and Poisson approximations, Amer. Math. Monthly, 101, 1, 48-54, (1994) · Zbl 0802.60019

[48] Tan, J.; Yang, S., Ruin probability about dual Poisson model with discrete time, Chinese J. Appl. Probab. Statist., 21, 3, 235-243, (2005), (in Chinese) · Zbl 1165.91422

[49] Zhang, H.; Li, B., Characterizations of discrete compound Poisson distributions, Comm. Statist. Theory Methods, (2014), (in press)

[50] Zygmund, A., Trigonometric series, (2002), Cambridge University Press Cambridge · Zbl 1084.42003