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Tropical covers of curves and their moduli spaces. (English) Zbl 1318.14060

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Let $\mathcal{L}$ be the abstract curve that corresponds to a generic tropical line in the tropical projective plane, i.e. a curve with one vertex that one denotes by c and three ends adjacent to c called u, v and w. Let $h:\Gamma\to\mathcal{L}$ be a cover of degree d. The weights of the ends mapping to u, v and w give rise to partitions Δ_u, Δ_v and Δ_w of d, and the triple $\Delta=(\Delta_u,\Delta_v,\Delta_w)$ is called the ramification profile of h. Now the authors introduce $\mathrm{M}_g^{\mathrm{trop}}(\mathcal{L},\Delta)$, the moduli space of tropical covers of $\mathcal{L}$ of genus g with ramification profile Δ.

The tropical branch map on $\mathrm{M}_g^{\mathrm{trop}}(\mathcal{L},\Delta)$ is $\mathrm{br}^{\mathrm{trop}}:\mathrm{M}_g^{\mathrm{trop}}(\mathcal{L},\Delta)\to\mathcal{L}^r$, $(h:\Gamma\to\mathcal{L})\mapsto(h(V_1),h(V_2),\dots,h(V_r))$, with $r:=\#\Delta+2g-2-d$ the total number of labels.

The main theorems are:

Theorem 2.15. The moduli space $\mathrm{M}_g^{\mathrm{trop}}(\mathcal{L},\Delta)$ is an abstract weighted polyhedral complex of pure dimension r.

Theorem 3.3. The degree of $\mathrm{br}^{\mathrm{trop}}$ is constant, called the tropical Hurwitz number $H_d^{g,\mathrm{trop}}(\Delta)$.

Theorem 3.6. The tropical Hurwitz numbers $H_d^{g,\mathrm{trop}}(\Delta)$ defined using tropical intersection theory equal their algebraic counterparts $H_d^g(\Delta)$.

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MSC:

14T05	Tropical geometry (MSC2010)
14N35	Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebro-geometric aspects)
51M20	Polyhedra and polytopes; regular figures, division of spaces

Keywords:
tropical geometry; Hurwitz numbers; covers of curves

Full Text: DOI arXiv

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