**Meijer, Henk**; **Akl, Selim G.**
**Optimal computation of prefix sums on a binary tree of processors.** (English) Zbl 0639.68032
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Given n numbers $a_0, a_1, ..., a_{n-1}$, it is required to compute all sums of the form $a_0 + a_1 + ... + a_i$, for $i = 0, 1, ..., n-1$. This problem arises in many applications and is trivial to solve sequentially in O(n) time. Besides its practical importance, the problem gains an additional theoretical interest in parallel computation. A technique known as recursive doubling allows all sums to be computed in O(log n) time on a model of computation where n processors communicate through an inverse perfect shuffle interconnection network.

In this paper we show how the problem can be solved on a simple network, namely a binary tree of processors. In addition, we show how to extend our solution to obtain an optimal-cost algorithm. The algorithm uses p processors and runs in $O((n/p) + \log p)$ time, for a cost of $O(n + p \log p)$. This cost is optimal when p log p= $O(n)$. Finally, two applications of our results are illustrated, namely job scheduling with deadlines and the knapsack problem.

**MSC:**

68M20 Performance evaluation, queueing, and scheduling in the context of computer systems
68N25 Theory of operating systems
68Q25 Analysis of algorithms and problem complexity
90C35 Programming involving graphs or networks



**Keywords:**

parallel computation; recursive doubling; model of computation; inverse perfect shuffle; binary tree of processors; optimal-cost algorithm; job scheduling; deadlines; knapsack problem

**Full Text:** DOI

**References:**

[1] S. G. Akl,Parallel Sorting Algorithms, Academic Press, Orlando, Florida (1985).

[2] E. Dekel and S. Sahni, Binary Trees and Parallel Scheduling Algorithms,IEEE Transactions on Computers C-32(3):307-315 (March 1983). · Zbl 0513.68031 · doi:10.1109/TC.1983.1676223

[3] F. E. Fich, New Bounds For Parallel Prefix Circuits,Proc. of the 15th Annual ACM Symposium on Theory of Computing, Boston, Massachusetts, pp. 100-109 (May 1983).

[4] P. M. Kogge, Parallel Solution of Recurrence Problems,IBM Journal of Research and Development, pp. 138-148 (March 1974). · Zbl 0307.65080

[5] P. M. Kogge and H. S. Stone, A Parallel Algorithm for the Efficient Solution of a General Class of Recurrence Equations,IEEE Transactions on Computers C-22(8):786-792 (August 1973). · Zbl 0262.68015 · doi:10.1109/TC.1973.5009159

[6] C. P. Kruskal, L. Rudolph, and M. Snir, The Power of Parallel Prefix,IEEE Transactions on Computers C-34 (10):965-968 (October 1985).

[7] R. E. Ladner and M. J. Fischer, Parallel Prefix Computation,Journal of the ACM 27(4):831-838 (October 1980). · Zbl 0445.68066 · doi:10.1145/322217.322232

[8] H. Meijer and S. G. Akl, Bit Serial Addition Trees and Their Applications,Proceedings of the Canadian Information Processing Society Congress '87, Winnipeg, Manitoba, pp. 319-322 (May 1987).

[9] J. Reif, Probabilistic Parallel Prefix Computation,Proceeding of the International Conference on Parallel Processing, Bellaire, Michigan, pp. 291-298 (August 1984).

[10] J. T. Schwartz, Ultracomputers,ACM Transactions on Programming Languages and Systems 2(4):484-521 (October 1980). · Zbl 0468.68027 · doi:10.1145/357114.357116

[11] H. S. Stone, (ed.),Introduction to Computer Architecture, Science Research Associates, Chicago, Illinois (1980). · Zbl 0322.68004

[12] R. Wagner and Y. Han, Parallel Algorithms For Bucket Sorting and the Data Dependent Prefix Problem,Proceedings of the Internatinal Conference on Parallel Processing, St. Charles, Illinois, pp. 924-930 (August 1986).

[13] R. P. Brent, The Parallel Evaluation of General Arithmetic Expressions,Journal of the ACM 21(2):201-206 (1974). · Zbl 0276.68010 · doi:10.1145/321812.321815

[14] E. Horowitz and S. Sahni,Fundamentals of Computer Algorithms, Computer Science Press, Potomac, Maryland (1978). · Zbl 0442.68022