The authors introduce the notion of pseudo-real principal G-bundle over a compact Kähler manifold X, equipped with an anti-holomorphic involution $\sigma_X : X \to X$ compatible with the Kähler structure. The group $G$ is a connected reductive affine algebraic group, defined over $\mathbb{C}$, equipped with a real form $\sigma_G : G \to G$.

Fix a maximal compact subgroup $K_G \subset G$ preserved by $\sigma_G$. The definition of pseudo-real principal G-bundle (see Definition 2.1 of the paper) depends on the choice of an element $c$ in the center of $K_G$ and which is fixed by $\sigma_G$. As the name suggests, this notion generalizes the notion of real principal G-bundle, which is indeed obtained if one takes $c = e$, the unit element of $G$.

The definitions of stable, semistable and polystable pseudo-real principal G-bundles are deduced, and the authors show that a pseudo-real principal G-bundle is semistable (respectively polystable) if and only if the underlying holomorphic principal G-bundle is semistable (respectively polystable). Moreover, it is proved a type of Donaldson-Uhlenbeck-Yau correspondence, namely that a pseudo-real principal G-bundle is polystable if and only if it admits an Einstein-Hermitian reduction of structure group to $K_G$. Finally, the relation with representations with the fundamental group of the corresponding real Kähler manifold $X$ acts by conjugation in this space, and it is shown that there is a natural bijective correspondence between elements of $\text{Hom}_e(\Gamma(X, x_0), K)$ and isomorphism classes of polystable pseudo-real principal $G$-bundles whose all rational characteristic classes of positive degree of the underlying topological principal $G$-bundle vanish.

All these results are then further generalized to the G-Higgs bundles setting, after introducing the definition of pseudo-real Higgs G-bundle. The statement of the results is a direct generalization of the ones presented in the previous paragraph, taking into account the presence of the Higgs field, and noting, in particular, that in the bijection with representations one has to replace representations in $K_G$ and $\tilde{K}$ by representations in $G$ and $\tilde{G} = G \rtimes (\mathbb{Z}/2\mathbb{Z})$.

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