zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Brzdęk, Janusz

Hyperstability of the Cauchy equation on restricted domains. (English) Zbl 1313.39037

Acta Math. Hung. 141, No. 1-2, 58-67 (2013).

The author extends classical results about Hyers-Ulam stability of the additive Cauchy equation from one normed space E_1 to another E_2. His maps are not required to be defined on all of E_1, but just on non-empty subsets X of $E_1 \setminus \{0\}$ with the following property: There exists a positive integer m_0 such that $x \in X \Rightarrow -x \in X$ and $nx \in X$ for all integers $n \geq m_0$.

Under these conditions his main result is: Let $c \geq 0$ and $p < 0$. Any map $g : X \to E_2$ satisfying

$$\|g(x+y) - g(x) - g(y)\| \leq c(\|x\|^p + \|y\|^p) \text{ whenever } x, y, x+y \in X,$$

is additive on X.

The corresponding result for $p \geq 0$ is not true, so $p < 0$ is essential.

The proof is based on the work by the author et al. [Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 74, No. 17, 6728–6732 (2011; Zbl 1236.39022)].

Reviewer: Henrik Stetkaer (Aarhus)

MSC:

39B82 Stability, separation, extension, and related topics for functional equations

39B52 Functional equations for functions with more general domains and/or ranges

Cited in **3** Reviews
Cited in **64** Documents

Keywords:

hyperstability; cocycle; Hyers-Ulam stability; additive Cauchy equation; normed space

Full Text: DOI

References:

[1] Aoki, T., On the stability of the linear transformation in Banach spaces, J. Math. Soc. Japan, 2, 64-66, (1950) · Zbl 0040.35501

[2] Baker, J. A., The stability of certain functional equations, Proc. Amer. Math. Soc., 112, 729-732, (1991) · Zbl 0735.39004

[3] Borelli Forti, C., Solutions of a non-homogeneous Cauchy equation, Radovi Mat., 5, 213-222, (1989) · Zbl 0697.39009

[4] Bourgin, D. G., Approximately isometric and multiplicative transformations on continuous function rings, Duke Math. J., 16, 385-397, (1949) · Zbl 0033.37702

[5] Bourgin, D. G., Classes of transformations and bordering transformations, Bull. Amer. Math. Soc., 57, 223-237, (1951) · Zbl 0043.32902

[6] Brzdęk, J.; Rassias, Th. M. (ed.); Tabor, J. (ed.), A note on stability of additive mappings, 19-22, (1994), Florida · Zbl 0844.39012

[7] Brzdęk, J.; Chudziak, J.; Páles, Zs., A fixed point approach to stability of functional equations, Nonlinear Anal., 74, 6728-6732, (2011) · Zbl 1236.39022

[8] Brzdęk, J.; Ciepliński, K., A fixed point approach to the stability of functional equations in non-Archimedean metric spaces, Nonlinear Anal., 74, 6861-6867, (2011) · Zbl 1237.39022

[9] L. Cădariu and V. Radu, Fixed point methods for the generalized stability of functional equations in a single variable, \textit{Fixed Point Theory Appl.}, \textbf{2008} (2008), Article ID 749392, 15 pages. · Zbl 0860.39037

[10] Ciepliński, K., Applications of fixed point theorems to the Hyers-Ulam stability of functional equations - a survey, Ann. Funct. Anal., 3, 151-164, (2012) · Zbl 1252.39032

[11] Davison, T. M. K.; Ebanks, B., Cocycles on cancellative semigroups, Publ. Math. Debrecen, 46, 137-147, (1995) · Zbl 0860.39037

[12] Ebanks, B., Generalized Cauchy difference functional equations, Aequationes Math., 70, 154-176, (2005) · Zbl 1079.39017

[13] Ebanks, B., Generalized Cauchy difference equations. II, Proc. Amer. Math. Soc., 136, 3911-3919, (2008) · Zbl 1206.39022

Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
© 2021 FIZ Karlsruhe GmbH

[14] B. Ebanks, P. Sahoo and W. Sander, \textit{Characterizations of Information Measures}, World Scientific (Singapore-New Jersey-London-Hong Kong, 1998). · Zbl 0923.94002

[15] Ebanks, B.; Kannappan, P. L.; Sahoo, P. K., Cauchy differences that depend on the product of arguments, Glasnik Mat., 27, 251-261, (1992) · Zbl 0780.39007

[16] Erdős, J., A remark on the paper "on some functional equations" by S. kurepa, Glasnik Mat.-Fiz. Astronom., 14, 3-5, (1959) · Zbl 0085.32903

[17] Fenyő, I.; Forti, G.-L., On the inhomogeneous Cauchy functional equation, Stochastica, 5, 71-77, (1981) · Zbl 0492.39002

[18] Forti, G.-L., Comments on the core of the direct method for proving Hyers-Ulam stability of functional equations, J. Math. Anal. Appl., 295, 127-133, (2004) · Zbl 1052.39031

[19] Forti, G.-L., Elementary remarks on Ulam-Hyers stability of linear functional equations, J. Math. Anal. Appl., 328, 109-118, (2007) · Zbl 1111.39026

[20] Gajda, Z., On stability of additive mappings, Int. J. Math. Math. Sci., 14, 431-434, (1991) · Zbl 0739.39013

[21] Gilanyi, A.; Kaiser, Z.; Páles, Zs., Estimates to the stability of functional equations, Aequationes Math., 73, 125-143, (2007) · Zbl 1121.39029

[22] Gselmann, E., Hyperstability of a functional equation, Acta Math. Hungar., 124, 179-188, (2009) · Zbl 1212.39044

[23] Hyers, D. H., On the stability of the linear functional equation, Proc. Natl. Acad. Sci. U.S.A., 27, 222-224, (1941) · Zbl 0061.26403

[24] D. H. Hyers, G. Isac and Th. M. Rassias, \textit{Stability of Functional Equations in Several Variables}, Birkhäuser (Boston, 1998). · Zbl 0907.39025

[25] Járai, A.; Maksa, Gy.; Páles, Zs., On Cauchy-differences that are also quasisums, Publ. Math. Debrecen, 65, 381-398, (2004) · Zbl 1071.39026

[26] Jessen, B.; Karpf, J.; Thorup, A., Some functional equations in groups and rings, Math. Scand., 22, 257-265, (1968) · Zbl 0183.04004

[27] S.-M. Jung, \textit{Hyers-Ulam-Rassias Stability of Functional Equations in Nonlinear Analysis}, Springer Optimization and Its Applications \textbf{48}, Springer (New York-Dordrecht-Heidelberg-London, 2011). · Zbl 1221.39038

[28] Jung, S.-M.; Kim, T.-S., A fixed point approach to the stability of the cubic functional equation, Bol. Soc. Mat. Mexicana, 12, 51-57, (2006) · Zbl 1133.39028

[29] Jung, S.-M.; Kim, T.-S.; Lee, K.-S., A fixed point approach to the stability of quadratic functional equation, Bull. Korean Math. Soc., 43, 531-541, (2006) · Zbl 1113.39031

[30] Jung, Y.-S.; Chang, I.-S., The stability of a cubic functional equation and fixed point alternative, J. Math. Anal. Appl., 306, 752-760, (2005) · Zbl 1077.39026

[31] M. Kuczma, \textit{An Introduction to the Theory of Functional Equations and Inequalities, Cauchy's Equation and Jensen's Inequality}, 2nd ed., Birkhäuser (2009). · Zbl 1221.39041

[32] Maksa, Gy., The stability of the entropy of degree alpha, J. Math. Anal. Appl., 346, 17-21, (2008) · Zbl 1149.39024

[33] Maksa, Gy.; Páles, Zs., Hyperstability of a class of linear functional equations, Acta Math. Acad. Paedag. Nyíregyháziensis, 17, 107-112, (2001) · Zbl 1004.39022

[34] Mirzavaziri, M.; Moslehian, M. S., A fixed point approach to stability of a quadratic equation, Bull. Braz. Math. Soc., New Series, 37, 361-376, (2006) · Zbl 1118.39015

[35] Radu, V., The fixed point alternative and the stability of functional equations, Fixed Point Theory, 4, 91-96, (2003) · Zbl 1051.39031

[36] Rassias, Th. M., On the stability of the linear mapping in Banach spaces, Proc. Amer. Math. Soc., 72, 297-300, (1978) · Zbl 0398.47040

[37] Rassias, Th. M., On a modified Hyers-Ulam sequence, J. Math. Anal. Appl., 158, 106-113, (1991) · Zbl 0746.46038

[38] Rassias, Th. M.; Semrl, P., On the behavior of mappings which do not satisfy Hyers-Ulam stability, Proc. Amer. Math. Soc., 114, 989-993, (1992) · Zbl 0761.47004

[39] P. K. Sahoo and P. Kannappan, \textit{Introduction to Functional Equations}, CRC Press (Boca Raton-London-New York, 2011). · Zbl 1223.39012

[40] Sikorska, J., On a direct method for proving the Hyers-Ulam stability of functional equations, J. Math. Anal. Appl., 372, 99-109, (2010) · Zbl 1198.39039

[41] Skof, F.; Rassias, Th. M. (ed.); Tabor, J. (ed.), On the stability of functional equations on a restricted domain and related topics, 141-151, (1994), Palm Harbor · Zbl 0844.39006

[42] S. M. Ulam, \textit{Problems in Modern Mathematics}, Science Editions, John-Wiley \& Sons Inc. (New York, 1964). · Zbl 0137.24201

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.