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Small scale creation for solutions of the incompressible two-dimensional Euler equation. (English) Zbl 1304.35521

Ann. Math. (2) 180, No. 3, 1205-1220 (2014).

The purpose of this article is to construct a two-dimensional Euler equation on the unit disc with smooth initial data, such that the solution has gradient of double exponential growth. The proofs use, among other methods, the Biot-Savart law, and the Green function.

Reviewer: Thomas Ernst (Uppsala)

MSC:

35Q31 Euler equations

Keywords:

double exponential growth; hyperbolic flow; small scale creation; two-dimensional incompressible flow; vorticity gradient growth

Full Text: DOI

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