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**A fractional order epidemic model for the simulation of outbreaks of influenza A(H1N1).**

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Summary: In this paper, we propose a nonlinear fractional order model in order to explain and understand the outbreaks of influenza A(H1N1). In the fractional model, the next state depends not only upon its current state but also upon all of its historical states. Thus, the fractional model is more general than the classical epidemic models. In order to deal with the fractional derivatives of the model, we rely on the Caputo operator and on the Grünwald–Letnikov method to numerically approximate the fractional derivatives. We conclude that the nonlinear fractional order epidemic model is well suited to provide numerical results that agree very well with real data of influenza A(H1N1) at the level population. In addition, the proposed model can provide useful information for the understanding, prediction, and control of the transmission of different epidemics worldwide.

**MSC:**

92D30 Epidemiology
34K37 Functional-differential equations with fractional derivatives
65L05 Numerical methods for initial value problems



**Keywords:**

epidemic models; fractional order model; influenza A(H1N1); Grünwald-Letnikov method

**Full Text:** DOI

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