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A numerical analysis of the Cahn-Hilliard equation with non-permeable walls. (English)

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This article is interested in the numerical analysis of the Cahn-Hilliard equation in a bounded domain with non-permeable walls equipped with dynamic-type boundary conditions. The considered dynamic-type boundary conditions describe the interactions of a binary material with the wall. The equation is semi-discretized using a finite element method for the space variables and error estimates between the exact and the approximate solution are obtained. The stability estimate is obtained using the backward Euler time difference formula. Numerical simulations supporting the theoretical results are presented.

Reviewer: Petr Sváček (Praha)

MSC:

- 65M20 Method of lines for initial value and initial-boundary value problems involving PDEs
- 65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs
- 65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
- 65M15 Error bounds for initial value and initial-boundary value problems involving PDEs
- 35Q35 PDEs in connection with fluid mechanics

Cited in 17 Documents

Keywords:

Cahn-Hilliard equation; finite element method; numerical examples; semidiscretization; non-permeable walls; error estimates; stability

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