

Żaba, Mariusz; Garbaczewski, Piotr

Solving fractional Schrödinger-type spectral problems: Cauchy oscillator and Cauchy well.

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The authors present the results of a computer-assisted route to obtain the eigenvalues and the eigenfunctions of the 1D Cauchy-Schrödinger operator $H = (-\Delta)^{1/2} + V$, V being a local potential. The Cauchy oscillator (which also has an analytical solution) and Cauchy finite well spectral problems are mainly envisaged. The algorithms employed are a non-local version of Strang's splitting method, which is based on Trotter product formula.

Reviewer: [Mihai Pascu \(Bucureşti\)](#)

MSC:

- [81Q05](#) Closed and approximate solutions to the Schrödinger, Dirac, Klein-Gordon and other equations of quantum mechanics
- [81Q10](#) Selfadjoint operator theory in quantum theory, including spectral analysis
- [35S05](#) Pseudodifferential operators as generalizations of partial differential operators
- [35R11](#) Fractional partial differential equations

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