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Blow-up criterion of smooth solutions for the Boussinesq equations. (English) Zbl 1300.35109
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Summary: This paper is dedicated to the study the Boussinesq equations with fractional dissipation in n -dimensions ($n \geq 2$). We obtain a blow-up criterion for the local in time classical solution to the Boussinesq equations with arbitrarily small fractional powers of the Laplacian. Therefore our results significantly improve the previous works and are more closer to the resolution of the well-known regularity criterion issue on the inviscid Boussinesq equations.

MSC:

- 35Q35 PDEs in connection with fluid mechanics
- 35B65 Smoothness and regularity of solutions to PDEs
- 76D03 Existence, uniqueness, and regularity theory for incompressible viscous fluids
- 35B44 Blow-up in context of PDEs
- 26A33 Fractional derivatives and integrals
- 35R11 Fractional partial differential equations

Cited in 17 Documents

Keywords:

Boussinesq equations; fractional dissipation; blow-up criterion

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