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The linearization principle for a free boundary problem for viscous, capillary incompressible fluids. (English) Zbl 1300.35077

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Summary: The free boundary problem associated with a viscous incompressible surface wave subject to the capillary force on a free upper surface and the Dirichlet boundary condition on a fixed bottom surface is considered. In the spatially periodic case, a general linearization principle is proved, which gives, for sufficiently small perturbations from a linearly stable stationary solution, the existence of a global solution of the associated system and the exponential convergence of the latter to the stationary one. The convergence of the velocity, the pressure, and the free boundary is proved in anisotropic Sobolev-Slobodetskii spaces, and then a suitable change of variables is performed to state the problem in a fixed domain. This linearization principle is applied to the study of the rest state's stability in the case of general potential forces.

MSC:

35Q30 Navier-Stokes equations
76D05 Navier-Stokes equations for incompressible viscous fluids
76B45 Capillarity (surface tension) for incompressible inviscid fluids
35R35 Free boundary problems for PDEs

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free boundary problem; capillary force

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