

**Botur, Michal; Chajda, Ivan; Halaš, Radomír; Kühn, Jan; Paseka, Jan**

**Algebraic methods in quantum logic.** (English) [Zbl 1314.03003](#)

Olomouc: Palacký University, Faculty of Science (ISBN 978-80-244-4166-5/pbk). viii, 195 p. (2014).

The monograph (based on recent results of the authors) is devoted to the study of various algebraic structures that model non-classical logics, in particular MV-algebras, lattice effect algebras, commutative basic algebras, and non-associative BL-algebras (generalizations of Hájek's BL-logics). MV-algebras are studied in Chapter 2. A direct proof of Di Nola's representation theorem (and its extensions) is presented using Farkas' lemma and the finite embedding theorem. A new proof of the completeness of the Łukasiewicz axioms is obtained as a by-product. It is proved that a tense semisimple MV-algebra is induced by a time frame. A tense MV-algebra is an MV-algebra with a couple of unary operators expressing universal time quantifiers. Lattice effect algebras are studied in Chapter 3. Finitely generated varieties of distributive lattice effect algebras are axiomatized and the free  $n$ -generator algebras in these varieties are described. Tense operators are constructed.

Chapter 4 is devoted to non-associative logics. A subdirectly irreducible commutative basic algebra that is not an MV-algebra is constructed for an arbitrary infinite cardinality. Basic properties of states on commutative basic algebras are presented. It is shown that the class of non-associative BL-algebras forms a variety generated by non-associative  $t$ -norms. The last chapter deals with state-operators. Characterizations of subdirectly irreducible state BL-algebras and subdirectly irreducible state-morphism BL-algebras are presented. A general theory of state-morphism algebras is given. Generators of varieties of state-morphism algebras are described, in particular for BL-algebras, MTL-algebras, non-associative BL-algebras and pseudo MV-algebras.

Reviewer: [Josef Tkadlec \(Praha\)](#)

**MSC:**

- [03-02](#) Research exposition (monographs, survey articles) pertaining to mathematical logic and foundations
- [03G12](#) Quantum logic
- [03G25](#) Other algebras related to logic
- [06D35](#) MV-algebras
- [06F25](#) Ordered rings, algebras, modules
- [08A55](#) Partial algebras
- [81P10](#) Logical foundations of quantum mechanics; quantum logic (quantum-theoretic aspects)

**Keywords:**

[MV-algebra](#); [tense MV-algebra](#); [lattice effect algebra](#); [commutative basic algebra](#); [non-associative BL-algebra](#); [state-morphism](#)