Gölgeleyen, Fikret; **Yamamoto, Masahiro**
Stability of inverse problems for ultrahyperbolic equations. (English) Zbl 1335.35295
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The authors consider inverse problems of determining a coefficient or a source term in an ultrahyperbolic equation

$$\Delta_y u(x,y) - \Delta_x u(x,y) - p(x,y')u(x,y) = F(x,y),$$

where $x=(x_1,\dots,x_n)\in\mathbb{R}^n$, $y=(y_1,\dots,y_m)\in\mathbb{R}^m$, $y'=(y_2,\dots,y_m)\in\mathbb{R}^{m-1}$, $\Delta_x=\sum_{i=1}^n\partial^2_{x_i}$, $\Delta_y=\sum_{j=1}^m\partial^2_{y_j}$, by some lateral boundary data.

Consider the bounded domain $D\subseteq\mathbb{R}^n$ with smooth boundary ∂D, $T>0$, $T_1>0$, $G(t,T_1)=\{y\in\mathbb{R}^m;|y_1|<T,|y'|<T_1\}$, $G'(T,T_1)\cap\{y_1=0\}$, $\nu(x)=(\nu_1(x),\dots,\nu_n(x))$, the unit outward normal vector to $\partial D, \partial_\nu u=(\nabla_x u,\nu)$, $\nabla_x=(\partial_{x_1},\dots,\partial_{x_n})$, $\Gamma\subseteq\partial D$, $\partial D_+=\{x\in\partial D;((x-x_0,\nu)\geq 0\}$, with $(\cdots)$ being the scalar product in $\mathbb{R}^n$ or $\mathbb{R}^m$. The authors consider the system

(1) $\mathrm{Au}=\Delta_y u(x,y)-\Delta_x u(x,y)-p(x,y')u(x,y)=f(x,y')R(x,y)$, $(x,y)\in D\times G(T,T_1)$,

(2) $u(x,0,y')=\partial_{y_1}u(x,0,y')=0$, $(x,y')\in D\times G'(T,T_1)$,

(3) $u(x,y)=0$, $(x,y)\in\Gamma\times G(T,T_1)$,

and they use the normed spaces $(S_i,\|\cdot\|_i)$, $1\leq i\leq 10$.

The authors consider the following hypotheses: $M>0$ is fixed, $f\in S_1=L^2(D\times G')$, $p\in S_2=L^\infty(D\times G')$, $\|p\|_2\leq M$, $R\in H^1(-T,T;S_2)$, $\|\partial_{y_1}R\|_3\leq M$, where $S_3=L^2(-T,T;S_2)$, $\|f\|_1\leq M$, $\|\partial_{y_1}u\|_4\leq M$, where $S_4=H^2(D\times G)$, $(\exists r_0>0)$ $(\forall(x,y')\in D\times G')(|R(x,0,y')|\geq r_0)$, $\max\{|x-x_0|;x\in\overline{D}\}<\sqrt{\beta T^2+\delta^2}$, where $0<\beta<1$, $\delta>0$ and $x_0\notin\overline{D}$, $\partial D\cap\{|x-x_0|\geq\delta\}\subseteq\Gamma$. They denote

$$\Omega(\delta)=\{(x,y)\in D\times G(T,T_1);|x-x_0|^2-\beta|y|^2>\delta^2\},$$
$$\Omega'(\delta)=\Omega(\delta)\cap\{y_1=0\}.$$

They prove that, for any $\delta_1>\delta$, there exist $C>0$ and $\theta\in(0,1)$, depending on M and r_0, such that $\|f\|_5\leq C\|\partial_\nu\partial_{y_1}u\|_6^\theta$, where $S_5=L^2(\Omega'(\delta_1))$ and $S_6=L^2(\Gamma\times G)$.

The authors consider (1), (2), (3) in $D\times G(T,2T)$, $u=0$ on $\partial D\times G(T,2T)$, $\|\partial^k_{y_1}u\|_7\leq M$, $k\in\{1,2\}$, $T>\frac{1}{\sqrt{\beta}}\max\{|x-x_0|;x\in\overline{D}\}$, $\|\partial^k_{y_1}R\|_8\leq M$, $k\in\{1,2\}$, $|R(x,0,y')|=0$, $x\in\overline{D}$, $|y'|\leq 2T$, where $S_7=H^2(D\times G(T,2T))$, $S_8=L^2(-T,T;L^2(D\times\{|y'|<2T\}))$, and prove that, for any $\varepsilon>0$, there exist constants $C>0$ and $\theta\in(0,1)$, depending on ε, M, x_0, such that

$$\|f\|_9\leq C\sum_{k=1}^2\|\partial_\nu\partial^k_{y_1}u\|_{10},$$

where $S_9=L^2(D\times\{|y'|<T-\varepsilon\})$, $S_{10}=L^2(\partial D_+\times G(T,2T))$.

Finally, they prove Hölder estimates which are global and local and the key tool is the Carleman estimate.

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MSC:

35R30 Inverse problems for PDEs
35A25 Other special methods applied to PDEs
35B35 Stability in context of PDEs

Keywords:

ultrahyperbolic equation; inverse problem; stability; Carleman estimate

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