Narang, T. D.
A characterization of strictly convex metric linear spaces. (English) Zbl 0634.41026

The metric linear space \((E,d)\) is said to be strictly convex if
\[ d(x,0) \leq r, \quad d(y,0) \leq r \quad \text{imply} \quad d(x + y/2, 0) < r \]
for all \(x, y \in E\) and \(r\) is any positive real number. A subset \(G\) of \((E,d)\) is said to be semi-Chebyshev if each element of \(E\) has at most one best approximation in \(G\). It is proved that a metric linear space \((E,d)\) is strictly convex if and only if all convex subsets of \(E\) are semi-Chebyshev.

Reviewer’s note: according to the results of G. Albinus [Math. Nachr. 37, 177-196 (1968; Zbl 0157.439)] if the real metric linear space \((E,d)\) is strictly convex, then the space \(E\) is locally convex, a continuous norm on \(E\) exists and the metric \(d\) is norm-like.

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MSC:

41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)
41A52 Uniqueness of best approximation
41A50 Best approximation, Chebyshev systems

Keywords:

semi-Chebyshev subsets

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