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Euler-Heisenberg Lagrangians and asymptotic analysis in 1+1 QED. Part I: two-loop. (English) Zbl 1294.81301

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Summary: We continue an effort to obtain information on the QED perturbation series at high loop orders, and particularly on the issue of large cancellations inside gauge invariant classes of graphs, using the example of the l – loop N – photon amplitudes in the limit of large photons numbers and low photon energies. As was previously shown, high-order information on these amplitudes can be obtained from a nonperturbative formula, due to Affleck et al., for the imaginary part of the QED effective Lagrangian in a constant field. The procedure uses Borel analysis and leads, under some plausible assumptions, to a number of nontrivial predictions already at the three-loop level. Their direct verification would require a calculation of this 'Euler-Heisenberg Lagrangian' at three-loops, which seems presently out of reach. Motivated by previous work by Dunne and Krasnansky on Euler-Heisenberg Lagrangians in various dimensions, in the present work we initiate a new line of attack on this problem by deriving and proving the analogous predictions in the simpler setting of 1+1 dimensional QED. In the first part of this series, we obtain a generalization of the formula of Affleck et al. to this case, and show that, for both Scalar and Spinor QED, it correctly predicts the leading asymptotic behaviour of the weak field expansion coefficients of the two loop Euler-Heisenberg Lagrangians.

MSC:

81V10	Electromagnetic interaction; quantum electrodynamics
81T16	Nonperturbative methods of renormalization applied to problems in quantum field theory
81T18	Feynman diagrams
81V80	Quantum optics
70S05	Lagrangian formalism and Hamiltonian formalism in mechanics of particles and systems

Keywords:

field theories in lower dimensions; nonperturbative effects

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