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Representation of infinite-dimensional forward price models in commodity markets. (English)

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Summary: We study the forward price dynamics in commodity markets realised as a process with values in a Hilbert space of absolutely continuous functions defined by *D. Filipović* [Lect. Notes. Math. 1760, 134 p. (2001; Zbl 1008.91038)]. The forward dynamics are defined as the mild solution of a certain stochastic partial differential equation driven by an infinite-dimensional Lévy process. It is shown that the associated spot price dynamics can be expressed as a sum of Ornstein-Uhlenbeck processes, or more generally, as a sum of certain stationary processes. These results link the possibly infinite-dimensional forward dynamics to classical commodity spot models. We continue with a detailed analysis of multiplication and integral operators on the Hilbert spaces and show that Hilbert-Schmidt operators are essentially integral operators. The covariance operator of the Lévy process driving the forward dynamics and the diffusion term can both be specified in terms of such operators, and we analyse in several examples the consequences on model dynamics and their probabilistic properties. Also, we represent the forward price for contracts delivering over a period in terms of an integral operator, a case being relevant for power and gas markets. In several examples, we reduce our general model to existing commodity spot and forward dynamics.

MSC:

60H15	Stochastic partial differential equations (aspects of stochastic analysis)
60G51	Processes with independent increments; Lévy processes
60G10	Stationary stochastic processes
91G80	Financial applications of other theories
47B10	Linear operators belonging to operator ideals (nuclear, p-summing, in the Schatten-von Neumann classes, etc.)
47G10	Integral operators
46E35	Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems

Keywords:

forward price models; commodity markets; stochastic partial differential equation; Lévy process; infinite-dimensional stochastic processes; Ornstein-Uhlenbeck processes; stationary processes; Heath-Jarrow-Morton approach; Hilbert space; Hilbert-Schmidt operators; integral operators

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