Coffman, Adam

Weighted projective spaces and a generalization of Eves' theorem. (English) Zbl 1296.51034

J. Math. Imaging Vis. 48, No. 3, 432-450 (2014).

Let $\mathcal{S}$ be a finite set of signed lengths of directed segments in the projectively extended (real) Euclidean plane. The theorem of Howard H. Eves (1911–2004) is applicable to $\mathcal{S}$ and yields ratios (that are elements of the real projective line $\mathbb{R}P^1$) being projectively invariant; a special case is the cross ratio. Avoiding any notion of distance, the author develops a generalization of Eves' theorem (gEt) in terms of linear algebra and projective geometry. In the gEt $\mathbb{R}P^1$ is replaced with a weighted projective space $\mathbb{K}P(p)$ where $\mathbb{K}$ is a commutative field and $p = (p_0, \ldots, p_n)$ ("weight vector"). Configurations to which gEt applies are described by a weighting, coloring, and indexing scheme. To each configuration $\mathcal{S}$ of points of $\mathbb{K}P^D$ that satisfies a certain condition depending on the weight vector p an element $E_p \in \mathbb{K}P(p)$ is assigned and E_p is invariant under "morphisms" of the configuration which generalize projective transformations.

Moreover, the author introduces the new notion of reconstructibility of a weighted projective space; "the two main results are that complex weighted projective spaces are all reconstructible, and that some real weighted projective spaces are not."

Reviewer: Rolf Riesinger (Wien)

MSC:

51N15	Projective analytic geometry
51A20	Configuration theorems in linear incidence geometry
14E05	Rational and birational maps
14N05	Projective techniques in algebraic geometry

Keywords:
invariant theory; weighted projective space; cross ratio; Peano space

Full Text: DOI arXiv

References:

[1] Adobe Illustrator CS5, Version 15.0.0 · Zbl 1246.97007

[2] Barnabei, M.; Brini, A.; Rota, G.-C., On the exterior calculus of invariant theory, J. Algebra, 96, 120-160, (1985) · Zbl 0585.15005 · doi:10.1016/0021-8693(85)90043-2

[3] Brill, J., On certain analogues of anharmonic ratio, Pure Appl. Math. Q., 29, 286-302, (1898) · Zbl 28.0471.01

[4] Brill, M.; Barrett, E., Closed-form extension of the anharmonic ratio to N-space, Comput. Vis. Graph. Image Process., 23, 92-98, (1983)· doi:10.1016/0734-189X(83)90055-5

[5] Clifford, W., Analytical metrics, Pure Appl. Math. Q., 7, (1865)

[6] Crapo, H.; Richter-Gebert, J.; White, N. (ed.), Automatic proving of geometric theorems, 167-196, (1995), Norwell · Zbl 1114.68532 · doi:10.1007/978-94-015-8402-9$_8$

[7] Delorme, C.: Espaces projectifs anisotropes. Bull. Soc. Math. Fr. \textbf{103}, 203-223 (1975). MR 0404277 (53 #8080a) · Zbl 0314.14016

[8] Dolgachev, I., Weighted projective varieties, Vancouver, BC, 1981, Berlin· doi:10.1007/BFb0101508

[9] Eves, H.: A Survey of Geometry. Allyn & Bacon, Boston (1972). Revised edn. MR 0322653 (48 #1015) · Zbl 0226.50001

[10] Frantz, M.: The most underrated theorem in projective geometry. Presentation at the 2011 MAA MathFest, Lexington, KY, August 6, 2011

[11] Frantz, M., A car crash solved—with a swiss army knife, Math. Mag., 84, 327-338, (2011) · Zbl 1246.97007 · doi:10.4169/math.mag.84.5.327

[12] Ore, Ø.: Number Theory and its History. McGraw-Hill, New York (1948). MR 0026059 (10,100b) · Zbl 0041.36805

[13] Poncelet, J.-V.: Traité des Propriétés Projectives des Figures, 1st edn. Bachelier, Paris (1822)

[14] Richter-Gebert, J.: Perspectives on Projective Geometry. Springer, Berlin (2011). MR 2791970 (2012e:51001) · Zbl 1214.51001 · doi:10.1007/978-3-642-17286-1

[15] Salmon, G.: Lessons Introductory to the Modern Higher Algebra, 5th edn. Chelsea, New York (1885)

[16] Shephard, G., Isomorphism invariants for projective configurations, Can. J. Math., 51, 1277-1299, (1999) · Zbl 0961.51017 · doi:10.4153/CJM-1999-058-8

