

**Allen, Benjamin; Nowak, Martin A.**

**Games on graphs.** (English) Zbl 1303.91040  
EMS Surv. Math. Sci. 1, No. 1, 113-151 (2014).

This paper offers an excellent survey over recent developments in the field of evolutionary game theory on graphs. Though the paper has been written from a biological perspective and thus includes some terms (neutral drift, fixation probability) with which non-biologists may be unfamiliar, it is accessible to readers from other sciences. An understanding of the paper requires some analytical familiarity with the mathematical tools used (such as pair approximation), but does not necessitate a previous knowledge of the literature.

The paper determines the conditions for long-term equilibria for symmetric  $2 \times 2$  games on finite and weighted graphs under the condition of weak selection, for the following updating rules: birth-death (BD), death-birth (DB), pairwise-comparison (PC), imitation (IM), birth-death with payoff affecting death (BD-D), and death-birth with payoff affecting death (DB-D).

For the simplified prisoner's dilemma (CC:  $b - c$ ; CD:  $-c$ ; DC:  $b$ ; and DD: 0), and BD, PC, and DB-D updating, cooperation is only favored if and only if self-interaction occurs and grants a payoff exceeding the cost  $c$ . In the case of DB and BD-D updating, the cooperative equilibrium does not require self-interaction and will prevail in the long-run (i.e. to cooperate has a higher fixation rate) if the condition  $b/c > \kappa$  is met, with  $\kappa$  being the Simpson degree.

The paper also extends results to nonzero mutation rates for BD and DB updating, as well as for cases in which the replacement graph and interaction graph of a game are not identical. At the end, the paper generally and shortly discusses asymmetric games, games with more than two players or strategies, and graphs with an endogenous topology but does not provide explicit results for these cases.

Reviewer: [Sebastian Ille \(Pisa\)](#)

**MSC:**

- [91A22](#) Evolutionary games
- [91A43](#) Games involving graphs
- [60J20](#) Applications of Markov chains and discrete-time Markov processes on general state spaces (social mobility, learning theory, industrial processes, etc.)
- [05C57](#) Games on graphs (graph-theoretic aspects)
- [91A05](#) 2-person games

Cited in **29** Documents

**Keywords:**

[evolutionary game theory on graphs](#); [symmetric  \$2 \times 2\$  games](#); [finite and weighted graphs](#); [updating rules](#); [cooperation](#)

**Full Text:** [DOI](#)

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