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Primitive stable representations of free Kleinian groups. (English) Zbl 1361.57025

Isr. J. Math. 199, Part B, 841-866 (2014).

Let F be a non-abelian free group of rank n . The authors give a complete criterion for a discrete faithful representation $\rho : F_n \rightarrow \mathrm{PSL}(2, \mathbb{C})$ to be primitive stable. Theorem 1.1 states that if ρ is a discrete faithful representation of F without parabolics then ρ is primitive stable. Theorem 1.2 states the following. Let ρ be a discrete, faithful and geometrically infinite representation with parabolics such that the non-cuspidal part M_0 of $M = \mathbb{H}^3/\rho(F)$ is the union of the relative compact core H and finitely many end neighbourhoods E_i facing $S_i \subset \partial H$. Then the representation ρ is primitive stable if and only if every parabolic curve is disc-busting, and every geometrically infinite end E_i has the ending lamination λ_i which is disc-busting on ∂H . These results answer *Y. N. Minsky's* conjectures [Isr. J. Math. 193, 47–70 (2013; Zbl 1282.57023)] about geometric conditions on $\mathbb{H}^3/\rho(F_n)$ regarding the primitive stability of ρ .

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MSC:

[57M60](#) Group actions on manifolds and cell complexes in low dimensions
[20F67](#) Hyperbolic groups and nonpositively curved groups
[20E05](#) Free nonabelian groups

Cited in **5** Documents

Keywords:

[Kleinian groups](#); [discrete faithful representations](#); [laminations](#); [Masur domain](#); [parabolic curve](#)

Full Text: [DOI](#)

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