

**Caffarelli, Luis**

**The homogenization of surfaces and boundaries.** (English) Zbl 1296.35030  
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The author briefly discusses the heuristics behind the structure of a couple of classical free-boundary problems for which some kind of homogenization asymptotics can be studied.

Reviewer: [Adrian Muntean \(Eindhoven\)](#)

**MSC:**

- [35J15](#) Second-order elliptic equations
- [35R60](#) PDEs with randomness, stochastic partial differential equations
- [35B27](#) Homogenization in context of PDEs; PDEs in media with periodic structure

**Keywords:**

[homogenization](#); [free boundaries](#)

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