Sznitman, Alain-Sol

On scaling limits and Brownian interlacements. (English) Zbl 1303.60022

Bull. Braz. Math. Soc. (N.S.) 44, No. 4, 555-592 (2013).

The author investigates the scaling limit of the field of occupation times of continuous time interlacements on $\mathbb{Z}^d$, $d \geq 3$. Let $\mathcal{L}^N$, $N \geq 1$, be the random measures on $\mathbb{R}^d$ given by the equation

$$\mathcal{L}^N = \frac{1}{dN^2} \sum_{x \in \mathbb{Z}^d} L_{x,u_N} \delta_{x/N},$$

where L_{x,u_N} is the field of occupation times of random interlacements at level u_N and $\{u_N\}_{N \geq 1}$ is a suitably chosen sequence of positive numbers.

The first main result states that, in the constant intensity regime ($u_N = d\alpha N^{d-2}$, $\alpha > 0$), the random measures $\mathcal{L}^N$ converges in distribution to $\mathcal{L}_\alpha$, as $N \to \infty$, where $\mathcal{L}_\alpha$ denotes the occupation-time measure of Brownian interlacements at level α.

The second main result states that, in the high intensity regime ($u_N N^{d-2} \to \infty$), convergence in distribution to the massless Gaussian free field holds for

$$\hat{\mathcal{L}}^N = \sqrt{\frac{d}{2N^{2-d}u_N}} (\mathcal{L}^N - \mathbb{E}[\mathcal{L}^N]).$$

At the end of paper, there is the scaling limit theorem considered using the isomorphism theorem due to the author [Electron. Commun. Probab. 17, Paper No. 9, 9 p. (2012; Zbl 1247.60135)] and it is applied to the case $d = 3$.

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MSC:

60F05 Central limit and other weak theorems
60J65 Brownian motion
60G60 Random fields
60J27 Continuous-time Markov processes on discrete state spaces

Keywords:

Brownian interlacements; scaling limits; massless Gaussian free field; isomorphism theorems

Full Text: DOI arXiv

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