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Frequency moments, L_q norms and Rényi entropies of general hypergeometric polynomials.

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Summary: The basic variables of the information theory of quantum systems (e.g., frequency or entropic moments, Rényi and Tsallis entropies) can be expressed in terms of L_q norms of general hypergeometrical polynomials. These polynomials are known to control the radial and angular parts of the wavefunctions of the quantum-mechanically allowed states of numerous physical and chemical systems. The computation of the L_q norms of these polynomials is presently an interesting issue *per se* in the theory of special functions; moreover, these quantities are closely related to the frequency moments and other information-theoretic properties of the associated Rakhmanov probability density. In this paper we calculate the unweighted and weighted L_q-norms ($q = 2k, k \in \mathbb{N}$) of general hypergeometric real orthogonal polynomials (Hermite, Laguerre and Jacobi) and some entropy-like integrals of Bessel polynomials, in terms of q and the parameters of the corresponding weight function by using their explicit expression and second order differential equation. In addition, the asymptotics ($q \to \infty$) of the unweighted L_q norms of the Jacobi polynomials is determined by the Laplace method.

MSC:

33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)
42C05 Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis

Keywords:

orthogonal polynomials; Hermite polynomials; Laguerre polynomials; Jacobi polynomials; Bessel polynomials; L_q-norms

Software:

DLMF

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