Gille, Philippe; Moret-Bailly, Laurent

Algebraic actions of arithmetic groups. (Actions algébriques de groupes arithmétiques.)

(French. English summary) Zbl 1317.14101
Skorobogatov, Alexei N. (ed.), Torsors, étale homotopy and applications to rational points. Lecture notes of mini-courses presented at the workshop "Torsors: theory and applications", Edinburgh, UK, January 10–14, 2011 and at the study group organised in Imperial College, London, UK in autumn 2010. Cambridge: Cambridge University Press (ISBN 978-1-107-61612-7/pbk; 978-1-139-52535-0/ebook). London Mathematical Society Lecture Note Series 405, 231-249 (2013).

In this article, the authors establish various finiteness results concerning the H^1_{fppf} of affine groups in p-adic and global characteristic-zero settings, using the modern theory of group schemes and torsors. Based on these results, they obtain the following generalization of a finiteness theorem of Platonov: Let S be a finite set of finite places of a number field F, and let A_S be the ring of S-integers in F. Consider a group A_S-scheme G and a flat A_S-scheme X of finite type equipped with a left G-action. Let $Z_0 \subset X$ be a closed A_S-subscheme which is flat over A_S, and let $\operatorname{loc}(Z_0)$ be the set of closed subschemes $Z \subset X$ which are $G(\overline{A_v})$-translates of Z at each finite place $v \notin S$, where $\overline{A_v}$ stands for the ring of integers in $\overline{F_v}$. Then $G(A_S)\backslash\operatorname{loc}(Z_0)$ is finite.

The geometric, i.e. equal characteristic $p > 0$ case is also discussed, and the finiteness theorem alluded to above requires stronger conditions.

For the entire collection see [Zbl 1277.14003].

Reviewer: Wen-Wei Li (Beijing)

MSC:

14L15 Group schemes
14G25 Global ground fields in algebraic geometry
14L05 Formal groups, p-divisible groups

Keywords:
arithmetic group; algebraic group; torsor