Kang, Hyunsuk; Lee, Lina; Zeager, Crystal
Comparison of invariant metrics. (English) Zbl 1293.32014
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Let $D \subset \mathbb{C}^n$ be a domain. By F_B^D and F_K^D we denote the pseudodifferential metrics of Bergman and Kobayashi, respectively.

The authors estimate precisely the quantity $F_K^{\mathbb{C}\setminus\{0,1\}}$ and compare the Bergman differential metrics of the unit ball B_n in $\mathbb{C}^n$ and the ring domain $\Omega_r = \{z \in \mathbb{C}^n \mid r < |z| < 1\}$, for $r \in (0,1)$.

Here are the results:

Theorem 1. Let $p \in \mathbb{C} \setminus \{0,1\}$ and $\delta = \operatorname{dist}(p,0)$ and $\xi = 1$. Then we have for small enough δ

$$F_K^{\mathbb{C}\setminus\{0,1\}}(p,\xi) \approx \frac{1}{\delta \log(1/\delta)}.$$

Theorem 2. Let $p \in \Omega_r$ and $\xi \in T_p\Omega_r$ be a tangent vector such that $p \cdot \overline{\xi} = 0$. Then

$$F_B^{\Omega_r}(p,\xi) \lneqq F_B^{B_n}(p,\xi)$$

for all $p \in \Omega_r$.

The restriction $p \cdot \overline{\xi} = 0$ can be let away in dimension two, if p lies on the normal to a point on the inner boundary of Ω_r.

Theorem 3. If $n = 2$ and $p = (r + \varepsilon, 0)$ for small $\varepsilon > 0$, then we have for small enough r and for arbitrary $\xi \in \mathbb{C}^2$ that

$$F_B^{\Omega_r}(p,\xi) \lneqq F_B^{B_2}(p,\xi)$$

Reviewer: Gregor Herbort (Wuppertal)

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32F45 Invariant metrics and pseudodistances in several complex variables

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