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Ending laminations and Cannon-Thurston maps; with an appendix by Shubhabrata Das and Mahan Mj. (English) Zbl 1297.57040

Geom. Funct. Anal. 24, No. 1, 297-321 (2014).

The paper under review contributes to the theory of Cannon-Thurston maps and to the study of points of non-injectivity for such maps. The main result considers the Cannon-Thurston map for surface fundamental groups acting as Kleinian groups on hyperbolic 3-space, and shows that for this map the preimage of a point is either one point, or two ideal boundary points of a leaf of the Kleinian groups ending lamination, or the ideal boundary points of a complementary ideal polygon for the ending lamination. (This result is proved for closed surfaces in the main part of the paper and for cusped surfaces in an appendix, written by the author with Shubhabrata Das.)

To put the paper into context, let us say that Cannon-Thurston theory considers the question whether group homomorphisms $f : G \to H$ between hyperbolic groups can be extended to equivariant continuous maps $\partial_\infty f : \partial_\infty G \to \partial_\infty H$ between their ideal boundaries. *J. W. Cannon* and *W. P. Thurston* in their 1985 preprint, published in [Geom. Topol. 11, 1315–1355 (2007; Zbl 1136.57009)], considered the case of closed hyperbolic manifolds M that fiber over the circle with fiber S, and showed that an inclusion $H^2 = \widetilde{S} \to \widetilde{M} = H^3$ extends to an equivariant Peano curve $S^1 = \partial_\infty H^2 \to \partial_\infty H^3 = S^2$.

Since then their result has seen many generalizations, culminating in the work of the author [Ann. Math. (2) 179, No. 1, 1–80 (2014; Zbl 1301.57013)] which showed that for any discrete faithful representation $\rho : \pi_1 S \to PSL(2,\mathbb{C})$ of a surface group and any imbedding $i : S \to N = H^3/\rho(\pi_1 S)$ inducing a homotopy equivalence, the lifted embedding $\tilde{i} : H^2 = \widetilde{S} \to \widetilde{N} = H^3$ extends continuously to the ideal boundary.

The constructions of that paper are also at the heart of the proofs in the paper under review. In the former paper it had been shown that images of geodesic segments outside a large ball in $\widetilde{S}$ are contained in so-called hyperbolic ladders and in particular outside large balls in $\widetilde{N}$. (This proved the existence of the Cannon-Thurston map.) In the paper under review the structure of certain specific ladders, namely those ladders corresponding to bi-infinite geodesics whose endpoints are identified by the extension of $\tilde{i}$, is further analyzed. Two different arguments are given, one in Section 4 and one in the Appendix. The first approach takes a geodesic that is not contained in the ending lamination but with endpoints of the lifted geodesic identified by the Cannon-Thurston map, and constructs another geodesic with the same property but its ideal endpoints consisting of the two fixed points of a loxodromic element, thus yielding a contradiction. The second approach uses a result of *B. H. Bowditch* [Math. Z. 255, No. 1, 35–76 (2007; Zbl 1138.57020)] to show that the geodesics, whose endpoints (of the lifted geodesic) are identified by the Cannon-Thurston map, form a lamination which then necessarily has to be the ending lamination.

As an application it is proved that simply or doubly degenerate representations of closed surface groups in $PSL(2,\mathbb{C})$ are quasiconformally conjugate if the actions on their limit sets are conjugate. This was shown by *J. F. Brock* et al. [Ann. Math. (2) 176, No. 1, 1–149 (2012; Zbl 1253.57009)] under the assumption that the limit set is the whole sphere.

In a recent preprint ["Cannon-Thurston maps for Kleinian groups", arXiv:1002.0996], the author has generalized the results to arbitrary finitely generated Kleinian groups.

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20F67 Hyperbolic groups and nonpositively curved groups
20F65 Geometric group theory
22E40 Discrete subgroups of Lie groups

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