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Galois theory of quadratic rational functions. (English) Zbl 1316.11104

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Let K be a number field, $\bar{K}$ its algebraic closure and $G_K := \mathrm{Gal}(\bar{K}/K)$ its absolute Galois group. Let $\phi \in K(x)$ be a rational function of degree 2. For each $\alpha \in K$ we consider the tree T_α whose vertex set is the disjoint union $\bigsqcup_{n \geq 1} \phi^{-n}(\alpha)$ of the iterated preimages with edges between $\phi^{-n}(\alpha)$ and $\phi^{-n+1}(\alpha)$ defined by the action of ϕ. The elements of G_K commute with ϕ and so we have a homomorphism $\rho : G_K \to \mathrm{Aut}(T_\alpha)$ called the arboreal Galois representation attached to (ϕ, α). The object of the paper is to study the image G_∞ of ρ for particular functions ϕ.

If ϕ commutes with some $f \in \mathrm{PGL}_2(K)$ and $f(\alpha) = \alpha$, then the Galois action on T_α commutes with the action of f. Define $A_\phi := \{f \in \mathrm{PGL}_2(\bar{K}) \mid \phi \circ f = f \circ \phi\}$ and let $A_{\phi,\alpha}$ be the stabilizer of α in A_ϕ. Let $C_\infty \leq G_\infty$ be the centralizer of action of $A_{\phi,\alpha}$ on $\mathrm{Aut}(T_\alpha)$. The authors are interested in the following conjecture. (Conjecture 1.1): If ϕ is not post-critically finite (that is, the orbit of at least one critical point under ϕ is infinite), then $|C_\infty : G_\infty| < \infty$. In an earlier paper [J. Lond. Math. Soc., II. Ser. 78, No. 2, 523–544 (2008; Zbl 1193.37144)] the first author proved that this conjecture holds for two special polynomials. In the present paper it is proved in some other cases. For example, suppose that $K = \mathbb{Q}$, $\phi(x) = k(x^2+1)/x$ and $\alpha = 0$. Then it is shown that there exists an effectively computable set Σ of primes of natural density 0 in $\mathbb{Z}$ such that $G_\infty \cong C_\infty$ provided the valuations $v_p(k)$ are 0 for all $p \in \Sigma$. More concretely the authors show that if $k \in \mathbb{Z}$ then Conjecture 1.1 holds whenever $|k| < 10000$.

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MSC:

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37P15 Dynamical systems over global ground fields

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