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On the critical behavior of the magnetization in high-dimensional Ising models. (English)

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We derive rigorously general results on the critical behavior of the magnetization in Ising models, as a function of the temperature and the external field. For the nearest-neighbor models it is shown that in $d \geq 4$ dimensions the magnetization is continuous at T_c and its critical exponents take the classical values $\delta = 3$ and $\beta =$, with possible logarithmic corrections at $d = 4$. The continuity, and other explicit bounds, formally extend to $d > 3$. Other systems to which the results apply include long-range models in $d = 1$ dimension, with $1/|x - y|^\lambda$ couplings, for which 2/(λ-1) replaces d in the above summary. The results are obtained by means of differential inequalities derived here using the random current representation, which is discussed in detail for the case of a nonvanishing magnetic field.

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory
82B27 Critical phenomena in equilibrium statistical mechanics
82B05 Classical equilibrium statistical mechanics (general)

Keywords:

critical exponents; critical behavior of the magnetization in Ising models; logarithmic corrections; long-range models

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References:

[1] A. D. Sokal, A rigorous inequality for the specific heat of an Ising or? 4 ferromagnet.Phys. Lett. 71A:451-453 (1979).

[2] M. Aizenman, Geometric analysis of? 4 2 fields and Ising models. Parts I and II,Commun. Math. Phys. 86:1-48 (1982). · Zbl 0533.58034 · doi:10.1007/BF01205659

[3] M. Aizenman and R. Graham, On the renormalized coupling constant and the susceptibility in? 4 4 field theory and the Ising model in four dimensions,Nucl. Phys. B225[FS9]:261-288 (1983).· doi:10.1016/0550-3213(83)90053-6

[4] J. Fröhlich, On the triviality of?? d 2 theories and the approach to the critical point ind > 4 dimension,Nucl. Phys. B200[FS4]:281-296 (1982).· doi:10.1016/0550-3213(82)90088-8

[5] S. Coleman and E. Weinberg, Radiative correction as the origin of spontaneous symmetry breakdown,Phys. Rev. D7:1888 (1973).

[6] M. Aizenman, Rigorous studies of critical behavior. II,Statistical Physics and Dynamical Systems: Rigorous Results (Birkhäuser, Boston, 1986, to be published). · Zbl 0667.60102

[7] J. Fröhlich and A. D. Sokal, to be published.

[8] C. Newman. private communication.

[9] J. Fröhlich, B. Simon, and T. Spencer, Infrared bounds, phase transition and continuous symmetry breaking,Commun. Math. Phys. 50:79-85 (1976).· doi:10.1007/BF01608557

[10] A. D. Sokal, An alternate constructive approach to the? 3 4 quantum field theory, and a possible destructive approach to? 4 4 ,Annal. Inst. Henri Poincaré 37:317-398 (1982).

[11] M. Aizenman, Rigorous studies of critical behavior,Applications of Field Theory in Statistical Mechanics, L. Garrido, ed., Springer Lecture Notes in Physics (SpringerVerlag, New York, in press). · Zbl 0667.60102

[12] D. Brydges, J. Fröhlich, and T. Spencer, The random walk representation of classical spin systems and correlation inequalities,Commun. Math. Phys. 83:123-150 (1982).· doi:10.1007/BF01947075

[13] D. C. Brydges, J. Fröhlich, and A. D. Sokal, The random walk representation of classical spin systems and correlation inequalities. II. The skeleton inequalities,Commun. Math. Phys. 91:117-139 (1983).· doi:10.1007/BF01206055

[14] R. Fernández, J. Fröhlich, and A. D. Sokal, in preparation.

[15] A. D. Sokal, More inequalities for critical exponents,J. Stat. Phys. 25:25-56 (1981).· doi:10.1007/BF01008477

[16] R. B. Griffiths, C. A. Hurst, and S. Sherman, Concavity of magnetization of an Ising ferromagnet in a positive external field,J.

Math. Phys. 11:790 (1970).· doi:10.1063/1.1665211

[17] E. Brezin, J. C. Le Guillou, and J. Zinn-Justin, inPhase Transitions and Critical Phenomena, C. Domb and M. S. Green, eds. (Academic Press, London, New York, San Francisco, 1976).

[18] J. Lebowitz, private communication.

[19] R. B. Griffiths, Correlations in Ising ferromagnets. II. External magnetic fields,J. Math. Phys. 8:484-489 (1967).· doi:10.1063/1.1705220

[20] M. E. Fisher, Critical temperatures of anisotropic Ising lattices. II. General upper bounds,Phys. Rev. 162:480-485 (1967).· doi:10.1103/PhysRev.162.480

[21] R. Graham, Correlation inequalities for the truncated two-point function of an Ising ferromagnet,J. Stat. Phys. 29:177-183 (1982).· doi:10.1007/BF01020780

[22] A. D. Sokal, private communication; see G. Felder and J. Fröhlich, Intersection properties of simple random walks: A renormalization group approach,Commun. Math. Phys. 97:111-124 (1985). · Zbl 0573.60065 · doi:10.1007/BF01206181

[23] J. Fröhlich, R. Israel, E. H. Lieb, and B. Simon, Phase transitions and reflection positivity. I. General theory and long-range lattice models,Commun. Math. Phys. 62:1 (1978).· doi:10.1007/BF01940327

[24] B. Simon and R. B. Griffiths, The (? 4)2 field theory as a classical Ising model,Commun. Math. Phys. 33:145-164 (1973).· doi:10.1007/BF01645626