Curto, Raúl E.; Hwang, In Sung; Kang, Dong-O; Lee, Woo Young
**Subnormal and quasinormal Toeplitz operators with matrix-valued rational symbols.** (English) Zbl 1318.47031
Adv. Math. 255, 562-585 (2014).

In [Pac. J. Math. 223, No. 1, 95–111 (2006; Zbl 1125.47019)], *C.-X. Gu* et al. extended Cowen's theorem to block Toeplitz operators: For each $\Phi \in L^\infty_{M_n}$, let $\mathcal{E}(\Phi) := \{K \in H^\infty_{M_n} : \|K\|_\infty \leq 1 \text{ and } \Phi - K\Phi^* \in H^\infty_{M_n}\}$. Then $T_\Phi$ is hyponormal if and only if $\Phi$ is normal and $\mathcal{E}(\Phi)$ is nonempty. First, the authors establish *M. B. Abrahamse*'s theorem [Duke Math. J. 43, 597–604 (1976; Zbl 0332.47017)] for matrix-valued rational symbols. Let $\Phi \in L^\infty_{M_n}$ be a matrix-valued rational function having a "matrix pole", i.e., there exists $\alpha \in \mathbb{D}$ for which $\ker H_\Phi \subseteq (z-\alpha)H^2_{\mathbb{C}^n}$, where $H_\Phi$ denotes the Hankel operator with symbol $\Phi$. Then the authors prove that, if (i) $T_\Phi$ is hyponormal and (ii) $\ker[T^*_\Phi, T_\Phi]$ is invariant for $T_\Phi$, then $T_\Phi$ is normal. Hence, in particular, if $T_\Phi$ is subnormal, then $T_\Phi$ is normal. Next, the authors establish Amemiya-Ito-Wong's theorem [*I. Amemiya* et al., Proc. Am. Math. Soc. 50, 254–258 (1975; Zbl 0339.47019)] for matrix-valued rational symbols. They prove that every pure quasinormal Toeplitz operator with a matrix-valued rational symbol is unitarily equivalent to an analytic Toeplitz operator.

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**MSC:**

47B20 Subnormal operators, hyponormal operators, etc.
47B35 Toeplitz operators, Hankel operators, Wiener-Hopf operators
46J15 Banach algebras of differentiable or analytic functions, $H^p$-spaces
15A83 Matrix completion problems
30H10 Hardy spaces
47A20 Dilations, extensions, compressions of linear operators



**Keywords:**

Amemiya, Ito and Wong's theorem; subnormal; quasinormal; hyponormal

**Full Text:** DOI

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