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Lévy flights and nonlocal quantum dynamics. (English) Zbl 1284.81012
J. Math. Phys. 54, No. 7, 072103, 34 p. (2013).

Summary: We develop a fully fledged theory of quantum dynamical patterns of behavior that are non-locally induced. To this end we generalize the standard Laplacian-based framework of the Schrödinger picture quantum evolution to that employing nonlocal (pseudodifferential) operators. Special attention is paid to the Salpeter (here, $m \geq 0$) quasirelativistic equation and the evolution of various wave packets, in particular to their radial expansion in 3D. Foldy's synthesis of "covariant particle equations" is extended to encompass free Maxwell theory, which however is devoid of any "particle" content. Links with the photon wave mechanics are explored.

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MSC:

- 81P05** General and philosophical questions in quantum theory
- 60G51** Processes with independent increments; Lévy processes
- 35S05** Pseudodifferential operators as generalizations of partial differential operators
- 81Q40** Bethe-Salpeter and other integral equations arising in quantum theory
- 70H40** Relativistic dynamics for problems in Hamiltonian and Lagrangian mechanics
- 81V80** Quantum optics

Cited in **1** Review
Cited in **4** Documents

Full Text: [DOI](#) [arXiv](#)

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