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Distance between conjugate algebraic numbers in clusters. (English. Russian original)

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Summary: For integers $n \geq 2$ and $Q > 1$, the following class of integer polynomials is defined

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

such that

$$\mathcal{P}_n(Q) = \{P \in \mathbb{Z}[x] : \deg \deg P = n, H(P) \leq Q\}$$

, where $H = H(P) = \max_{0 \leq j \leq n} |a_j|$ is the height of P. Let $\alpha_1 \dots \alpha_n \in \mathbb{C}$, $\alpha_i \neq \alpha_j$ be the roots of P. A systematic study of the quantities $|\alpha_i - \alpha_j|$ for various conjugates algebraic numbers α_i and α_j has been done and also of the more general problem of clusters

$$M_k = \prod_{1 \leq i < j \leq k} |\alpha_i - \alpha_j|$$

Next, denote by $E(n, k)$ (resp. $E_{\text{irr}}(n, k)$) the infimum of real numbers δ for which the inequality

$$\prod_{1 \leq i < j \leq k} |\alpha_i - \alpha_j| \geq H(P)^{-\delta}$$

holds for each integer (resp. integer irreducible) polynomial P of degree n. In the present paper the authors obtain sharp estimates for $E_{\text{irr}}(n, k)$).

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11B83 Special sequences and polynomials
11J68 Approximation to algebraic numbers

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References:

[1] A. Baker, W. M. Schmidt, Proc. London Math. Soc. (3), 21 (1970), 1 – 11 · Zbl 0206.05801 · doi:10.1112/plms/s3-21.1.1
[2] В. Г. Спринджук, Проблема Малера в метрической теории чисел, Наука и техника, Минск, 1967 · Zbl 0168.29504
[3] K. Mahler, Michigan Math. J., 11:3 (1964), 257 – 262 · Zbl 0135.01702 · doi:10.1307/mmj/1028999140
[4] V. V. Beresnevich, V. I. Bernik, F. Goẗze, Compositio Math., 146:5 (2010), 1165 – 1179 · Zbl 1206.11091 · doi:10.1112/S0010437X10004860 · arxiv:0906.4286
[5] Y. Bugeaud, M. Mignotte, Proc. Edinb. Math. Soc. (2), 47:2 (2004), 553 – 556 · Zbl 1071.11016 · doi:10.1017/S0013091503000257
[6] Y. Bugeaud, M. Mignotte, Int. J. Number Theory, 6:3 (2010), 587 – 602 · Zbl 1205.11032 · doi:10.1142/S1793042110003083
[7] Y. Bugeaud, A. Dujella, Bull. Lond. Math. Soc., 43:6 (2011), 1239 – 1244 · Zbl 1273.11049 · doi:10.1112/blms/bdr085
[8] Y. Bugeaud, A. Dujella, Root Separation for Reducible Integer Polynomials, arXiv: math.NT/1306.2128v1
[9] В. В. Бересневич, В. И. Берник, Ф. Гетце, Докл. НАН Беларуси, 54:5 (2010), 21 – 23 · Zbl 1267.11026

paper as accurately as possible without claiming the completeness or perfect precision of the matching.