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A transformed time-dependent Michaelis-Menten enzymatic reaction model and its asymptotic stability. (English) Zbl 1311.92072

J. Math. Chem. 52, No. 1, 222-230 (2014).

Summary: The dynamic form of the Michaelis-Menten enzymatic reaction equations provide a time-dependent model in which a substrate S reacts with an enzyme E to form a complex C which in turn is converted into a product P and the enzyme E. In the present paper, we show that this system of four nonlinear equations can be reduced to a single nonlinear differential equation, which is simpler to solve numerically than the system of four equations. Applying the Lyapunov stability theory, we prove that the non-zero equilibrium for this equation is globally asymptotically stable, and hence that the non-zero steady-state solution for the full Michaelis-Menten enzymatic reaction model is globally asymptotically stable for all values of the model parameters. As such, the steady-state solutions considered in the literature are stable. We finally discuss properties of the numerical solutions to the dynamic Michaelis-Menten enzymatic reaction model, and show that at small and large time scales the solutions may be approximated analytically.

MSC:

92C40 Biochemistry, molecular biology
92C45 Kinetics in biochemical problems (pharmacokinetics, enzyme kinetics, etc.)

Keywords:

dynamic Michaelis-Menten model; nonlinear dynamics; stability; enzyme reactions

Full Text: DOI

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