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Fixed point theorems for discontinuous maps on a non-convex domain. (English)

Zbl 1283.91112
Metroeconomica 64, No. 3, 547-572 (2013).

Summary: This paper introduces economists to some fixed point theorems for discontinuous mappings with non-convex images on a non-convex domain. These theorems have recently been developed based on a new approach by mathematical economists and mathematicians. The new method of proof is first transformed into a sort of metatheorem, which is then used to obtain a set of necessary and sufficient conditions for a map to have a fixed point. Some fixed point theorems for discontinuous maps are then explained in more concrete cases. The formulations are intended for easier applications towards economic models involving discontinuity as well as non-convexity.

MSC:

91B50 General equilibrium theory
54H25 Fixed-point and coincidence theorems (topological aspects)

Keywords:

fixed point theorems; discontinuous maps; non-convex domain; non-convex images

Full Text: DOI

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