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Ground states of the 2D sticky disc model: fine properties and $N^{3/4}$ law for the deviation from the asymptotic Wulff shape. (English) Zbl 1292.82027

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Summary: We investigate ground state configurations for a general finite number N of particles of the Heitmann-Radin sticky disc pair potential model in two dimensions. Exact energy minimizers are shown to exhibit large microscopic fluctuations about the asymptotic Wulff shape which is a regular hexagon: There are arbitrarily large N with ground state configurations deviating from the nearest regular hexagon by a number of $\sim N^{3/4}$ particles. We also prove that for any N and any ground state configuration this deviation is bounded above by $\sim N^{3/4}$. As a consequence we obtain an exact scaling law for the fluctuations about the asymptotic Wulff shape. In particular, our results give a sharp rate of convergence to the limiting Wulff shape.

MSC:

82C22 Interacting particle systems in time-dependent statistical mechanics

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