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Permanental partition models and Markovian Gibbs structures. (English) Zbl 1302.82071

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Summary: We study both time-invariant and time-varying Gibbs distributions for configurations of particles into disjoint clusters. Specifically, we introduce and give some fundamental properties for a class of partition models, called *permanental partition models*, whose distributions are expressed in terms of the α-permanent of a similarity matrix parameter. We show that, in the time-invariant case, the permanental partition model is a refinement of the celebrated Pitman-Ewens distribution; whereas, in the time-varying case, the permanental model refines the Ewens cut-and-paste Markov chains [*H. Crane*, J. Appl. Probab. 48, No. 3, 778–791 (2011; Zbl 1235.60092)]. By a special property of the α-permanent, the partition function can be computed exactly, allowing us to make several precise statements about this general model, including a characterization of exchangeable and consistent permanental models.

MSC:

82C22 Interacting particle systems in time-dependent statistical mechanics

Keywords:

Boltzmann-Gibbs measure; canonical Gibbs ensemble; Markovian Gibbs structure; permanental partition model; α-permanent; permanental process

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