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**Spectra of random Hermitian matrices with a small-rank external source: the supercritical and subcritical regimes.** (English) Zbl 1302.82049

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Summary: Random Hermitian matrices with a source term arise, for instance, in the study of non-intersecting Brownian walkers and sample covariance matrices. We consider the case when the  $n \times n$  external source matrix has two distinct real eigenvalues:  $a$  with multiplicity  $r$  and zero with multiplicity  $n - r$ . The source is small in the sense that  $r$  is finite or  $r = \mathcal{O}(n^\gamma)$ , for  $0 < \gamma < 1$ . For a Gaussian potential, *S. Péché* [*Probab. Theory Relat. Fields* 134, No. 1, 127–173 (2006; [Zbl 1088.15025](#))] showed that for  $|a|$  sufficiently small (the subcritical regime) the external source has no leading-order effect on the eigenvalues, while for  $|a|$  sufficiently large (the supercritical regime)  $r$  eigenvalues exit the bulk of the spectrum and behave as the eigenvalues of the  $r \times r$  Gaussian unitary ensemble (GUE). We establish the universality of these results for a general class of analytic potentials in the supercritical and subcritical regimes.

**MSC:**

- [82B41](#) Random walks, random surfaces, lattice animals, etc. in equilibrium statistical mechanics  
[15B52](#) Random matrices (algebraic aspects)

Cited in 7 Documents

**Keywords:**

[random matrices](#); [universality](#); [asymptotics](#); [external sources](#); [eigenvalues](#); [Riemann-Hilbert problems](#)

**Full Text:** [DOI](#)

**References:**

- [1] Adler, M.; Delépine, J.; Moerbeke, P., Dyson's nonintersecting Brownian motions with a few outliers, *Commun. Pure Appl. Math.*, 62, 334-395, (2009) · [Zbl 1166.60048](#)
- [2] Adler, M.; Orantin, N.; Moerbeke, P., Universality of the Pearcey process, *Physica D*, 239, 924-941, (2010) · [Zbl 1189.82085](#)
- [3] Aptekarev, A.; Bleher, P.; Kuijlaars, A., Large  $n$  limit of Gaussian random matrices with external source, part II, *Commun. Math. Phys.*, 259, 367-389, (2005) · [Zbl 1129.82014](#)
- [4] Baik, J., Painlevé formulas of the limiting distributions for non-null complex sample covariance matrices, *Duke Math. J.*, 133, 205-235, (2006) · [Zbl 1139.33006](#)
- [5] Baik, J.; Wang, D., On the largest eigenvalue of a Hermitian random matrix model with spiked external source I. rank one case, *Int. Math. Res. Not.*, 2011, 5164-5240, (2011) · [Zbl 1233.15011](#)
- [6] Baik, J.; Wang, D., On the largest eigenvalue of a Hermitian random matrix model with spiked external source II. higher rank cases, *Int. Math. Res. Not.*, 2013, 3304-3370, (2013) · [Zbl 1315.15033](#)
- [7] Baik, J.; Ben Arous, G.; Péché, S., Phase transition of the largest eigenvalue for non-null complex sample covariance matrices, *Ann. Probab.*, 33, 1643-1697, (2005) · [Zbl 1086.15022](#)
- [8] Bertola, M.; Lee, S.Y., First colonization of a spectral outpost in random matrix theory, *Constr. Approx.*, 30, 225-263, (2009) · [Zbl 1169.05385](#)
- [9] Bertola, M.; Lee, S.Y., First colonization of a hard-edge in random matrix theory, *Constr. Approx.*, 31, 231-257, (2010) · [Zbl 1250.05123](#)
- [10] Bertola, M.; Lee, S.Y.; Mo, M., Mesoscopic colonization of a spectral band, *J. Phys. A*, 42, (2009) · [Zbl 1179.81070](#)
- [11] Bertola, M.; Buckingham, R.; Lee, S.Y.; Pierce, V., Spectra of random Hermitian matrices with a small-rank external source: the critical and near-critical regimes, *J. Stat. Phys.*, 146, 475-518, (2012) · [Zbl 1241.82035](#)
- [12] Bleher, P.; Kuijlaars, A., Random matrices with external source and multiple orthogonal polynomials, *Int. Math. Res. Not.*, 2004, 109-129, (2004) · [Zbl 1082.15035](#)
- [13] Bleher, P.; Kuijlaars, A., Large  $n$  limit of Gaussian random matrices with external source, part I, *Commun. Math. Phys.*, 252, 43-76, (2004) · [Zbl 1124.82309](#)
- [14] Bleher, P.; Kuijlaars, A., Large  $n$  limit of Gaussian random matrices with external source, part III: double scaling limit, *Commun. Math. Phys.*, 270, 481-517, (2007) · [Zbl 1126.82010](#)

- [15] Brézin, E.; Hikami, S., Correlations of nearby levels induced by a random potential, Nucl. Phys. B, 479, 697-706, (1996) · [Zbl 0925.82117](#)
- [16] Brézin, E.; Hikami, S., Spectral form factor in a random matrix theory, Phys. Rev. E, 55, 4067-4083, (1997) · [Zbl 1042.76025](#)
- [17] Brézin, E.; Hikami, S., Extension of level-spacing universality, Phys. Rev. E, 56, 264-269, (1997)
- [18] Brézin, E.; Hikami, S., Level spacing of random matrices in an external source, Phys. Rev. E, 58, 7176-7185, (1998)
- [19] Deift, P.: Orthogonal Polynomials and Random Matrices: A Riemann-Hilbert Approach. Am. Math. Soc., Providence (1998) · [Zbl 0997.47033](#)
- [20] Deift, P.; Kriecherbauer, T.; McLaughlin, K.; Venakides, S.; Zhou, X., Uniform asymptotics for polynomials orthogonal with respect to varying exponential weights and applications to universality questions in random matrix theory, Commun. Pure Appl. Math., 52, 1335-1425, (1999) · [Zbl 0944.42013](#)
- [21] Deift, P.; Kriecherbauer, T.; McLaughlin, K.; Venakides, S.; Zhou, X., Strong asymptotics of orthogonal polynomials with respect to exponential weights, Commun. Pure Appl. Math., 52, 1491-1552, (1999) · [Zbl 1026.42024](#)
- [22] Deimling, K.: Ordinary Differential Equations in Banach Spaces. Lecture Notes in Mathematics, vol. 596. Springer, Berlin (1977) · [Zbl 0361.34050](#)
- [23] Ercolani, N.; McLaughlin, K., Asymptotics of the partition function for random matrices via Riemann-Hilbert techniques and applications to graphical enumeration, Int. Math. Res. Not., 2003, 755-820, (2003) · [Zbl 1140.82307](#)
- [24] Fokas, A.; Its, A.; Kitaev, A., The isomonodromy approach to matrix models in 2D quantum gravity, Commun. Math. Phys., 147, 395-430, (1992) · [Zbl 0760.35051](#)
- [25] Kuijlaars, A.B.J.; McLaughlin, K.T.-R., Generic behavior of the density of states in random matrix theory and equilibrium problems in the presence of real analytic external fields, Commun. Pure Appl. Math., 53, 736-785, (2000) · [Zbl 1022.31001](#)
- [26] Mather, J., Stability of  $\mathbb{C}^n \rightarrow \mathbb{C}^n$  mappings. I. the division theorem, Ann. Math. (2), 87, 89-104, (1968) · [Zbl 0159.24902](#)
- [27] Mather, J., Stability of  $\mathbb{C}^n \rightarrow \mathbb{C}^n$  mappings. II. infinitesimal stability implies stability, Ann. Math. (2), 89, 254-291, (1969) · [Zbl 0177.26002](#)
- [28] McLaughlin, K., Asymptotic analysis of random matrices with external source and a family of algebraic curves, Nonlinearity, 20, 1547-1571, (2007) · [Zbl 1117.82310](#)
- [29] Mehta, M.: Random Matrices, 3rd edn. Pure and Applied Mathematics, vol. 142. Elsevier/Academic Press, Amsterdam/San Diego (2004) · [Zbl 1107.15019](#)
- [30] Péché, S., The largest eigenvalue of small rank perturbations of Hermitian random matrices, Probab. Theory Relat. Fields, 134, 127-173, (2006) · [Zbl 1088.15025](#)
- [31] Tracy, C.; Widom, H., Level-spacing distributions and the Airy kernel, Commun. Math. Phys., 159, 151-174, (1994) · [Zbl 0789.35152](#)
- [32] Zinn-Justin, P., Random Hermitian matrices in an external field, Nucl. Phys. B, 497, 725-732, (1997) · [Zbl 0933.82022](#)
- [33] Zinn-Justin, P., Universality of correlation functions of Hermitian random matrices in an external field, Commun. Math. Phys., 194, 631-650, (1998) · [Zbl 0912.15028](#)

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