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Kinetic theory of jet dynamics in the stochastic barotropic and 2D Navier-Stokes equations.

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Summary: We discuss the dynamics of zonal (or unidirectional) jets for barotropic flows forced by Gaussian stochastic fields with white in time correlation functions. This problem contains the stochastic dynamics of 2D Navier-Stokes equation as a special case. We consider the limit of weak forces and dissipation, when there is a time scale separation between the inertial time scale (fast) and the spin-up or spin-down time (large) needed to reach an average energy balance. In this limit, we show that an adiabatic reduction (or stochastic averaging) of the dynamics can be performed. We then obtain a kinetic equation that describes the slow evolution of zonal jets over a very long time scale, where the effect of non-zonal turbulence has been integrated out. The main theoretical difficulty, achieved in this work, is to analyze the stationary distribution of a Lyapunov equation that describes quasi-Gaussian fluctuations around each zonal jet, in the inertial limit. This is necessary to prove that there is no ultraviolet divergence at leading order, in such a way that the asymptotic expansion is self-consistent. We obtain at leading order a Fokker-Planck equation, associated to a stochastic kinetic equation, that describes the slow jet dynamics. Its deterministic part is related to well known phenomenological theories (for instance stochastic structural stability theory) and to quasi-linear approximations, whereas the stochastic part allows to go beyond the computation of the most probable zonal jet. We argue that the effect of the stochastic part may be of huge importance when, as for instance in the proximity of phase transitions, more than one attractor of the dynamics is present.

MSC:

82C40 Kinetic theory of gases in time-dependent statistical mechanics
82C31 Stochastic methods (Fokker-Planck, Langevin, etc.) applied to problems in time-dependent statistical mechanics

Keywords:

turbulence; geostrophic turbulence; non-equilibrium steady states; stochastic partial differential equations; stochastic averaging; stochastic 2D Navier Stokes equations; stochastic barotropic equations; atmosphere dynamics

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