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Measures of success in a class of evolutionary models with fixed population size and structure. (English) Zbl 1280.92045

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Summary: We investigate a class of evolutionary models, encompassing many established models of well-mixed and spatially structured populations. Models in this class have fixed population size and structure. Evolution proceeds as a Markov chain, with birth and death probabilities dependent on the current population state. Starting from basic assumptions, we show how the asymptotic (long-term) behavior of the evolutionary process can be characterized by probability distributions over the set of possible states. We then define and compare three quantities characterizing evolutionary success: fixation probability, expected frequency, and expected change due to selection. We show that these quantities yield the same conditions for success in the limit of low mutation rate, but may disagree when mutation is present. As part of our analysis, we derive versions of the *G.R. Price* equation [see Ann. Hum. Genet. 35, 485–490 (1972; Zbl 0231.92006)] and the replicator equation that describe the asymptotic behavior of the entire evolutionary process, rather than the change from a single state. We illustrate our results using the frequency-dependent Moran process and the birth-death process on graphs as examples. Our broader aim is to spearhead a new approach to evolutionary theory, in which general principles of evolution are proven as mathematical theorems from axioms.

MSC:

92D15 Problems related to evolution
60J20 Applications of Markov chains and discrete-time Markov processes on general state spaces (social mobility, learning theory, industrial processes, etc.)
37N25 Dynamical systems in biology

Keywords:

stochastics; axioms; fixation probability; evolutionary success; price equation

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